JOURNAL OF THE MECHANICAL BEHAVIOR OF BIOMEDICAL MATERIALS 39 (2014) 48–60

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Research Paper

A robust anisotropic hyperelastic formulation for the modelling of soft tissue

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ARTICLE INFO

Article history:
Received 18 March 2014
Received in revised form
19 June 2014
Accepted 24 June 2014
Available online 11 July 2014

Keywords:
Anisotropic
Hyperelastic
Incompressibility
Finite element
Artery
Stent

ABSTRACT

The Holzapfel–Gasser–Ogden (HGO) model for anisotropic hyperelastic behaviour of collagen fibre reinforced materials was initially developed to describe the elastic properties of arterial tissue, but is now used extensively for modelling a variety of soft biological tissues. Such materials can be regarded as incompressible, and when the incompressibility condition is adopted the strain energy Ψ of the HGO model is a function of one isotropic and two anisotropic deformation invariants. A compressible form (HGO-C model) is widely used in finite element simulations whereby the isotropic part of Ψ is decoupled into volumetric and isochoric parts and the anisotropic part of Ψ is expressed in terms of isochoric invariants. Here, by using three simple deformations (pure dilatation, pure shear and uniaxial stretch), we demonstrate that the compressible HGO-C formulation does not correctly model compressible anisotropic material behaviour, because the anisotropic component of the model is insensitive to volumetric deformation due to the use of isochoric anisotropic invariants. In order to correctly model compressible anisotropic behaviour we present a modified anisotropic (MA) model, whereby the full anisotropic invariants are used, so that a volumetric anisotropic contribution is represented. The MA model correctly predicts an anisotropic response to hydrostatic tensile loading, whereby a sphere deforms into an ellipsoid. It also computes the correct anisotropic stress state for pure shear and uniaxial deformations. To look at more practical applications, we developed a finite element user-defined material subroutine for the simulation of stent deployment in a slightly compressible artery. Significantly higher stress triaxiality and arterial compliance are computed when the full anisotropic invariants are used (MA model) instead of the isochoric form (HGO-C model).

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http://dx.doi.org/10.1016/j.jmbbm.2014.06.016
1751-6161/

Nomenclature

$\mathbf{I}$	identity tensor
Ψ	Helmholtz free-energy (strain-energy) function
Ψ_{vol}	volumetric contribution to the free energy
Ψ_{aniso}	anisotropic contribution to the free energy
$\overline{\Psi}_{\text{iso}}$	isotropic contribution to the isochoric free energy
$\overline{\Psi}_{\text{aniso}}$	anisotropic contribution to the isochoric free energy
$\boldsymbol{\sigma}$	Cauchy stress
$\boldsymbol{\sigma}'$	deviatoric Cauchy stress
q	von Mises equivalent stress
σ_{hyd}	hydrostatic (pressure) stress
$\mathbf{F}$	deformation gradient
J	determinant of the deformation gradient; local ratio of volume change
$\mathbf{C}$	right Cauchy–Green tensor
I_1	first invariant of $\mathbf{C}$
$I_{4,6}$	anisotropic invariants describing the deformation of reinforcing fibres
$\overline{\mathbf{F}}$	isochoric portion of the deformation gradient
$\overline{\mathbf{C}}$	isochoric portion of the right Cauchy–Green deformation tensor
$\overline{I}_1$	first invariant of $\overline{\mathbf{C}}$
$\overline{I}_{4,6}$	isochoric anisotropic invariants
$\mathbf{a}_{0i}$, i=4, 6	unit vector aligned with a reinforcing fibre in the reference configuration
$\mathbf{a}_i$, i=4, 6	updated (deformed) fibre direction ($=\mathbf{F}\mathbf{a}_{0i}$)
κ_0	isotropic bulk modulus
μ_0	isotropic shear modulus
k_i, i=1, 2	anisotropic material constants
ν	isotropic Poisson's ratio

Bold uppercase symbols represent second order tensors, bold lowercase symbols represent vectors and un-bold symbols represent scalars.

1. Introduction

The anisotropic hyperelastic constitutive model proposed by Holzapfel et al. (2000) (henceforth referred to as the HGO model) is used extensively to model collagen fibre-reinforced biological materials, even more so now that it has been implemented in several commercial and open-source Finite Element (FE) codes for the simulation of soft tissue elasticity.

The constitutive equation builds upon previously published transversely isotropic constitutive models (e.g. Weiss et al., 1996) and reflects the structural components of a typical biological soft tissue, hence its strain-energy density consists of two mechanically equivalent terms accounting for the anisotropic contributions of the reinforcing fibre families, in addition to a term representing the isotropic contribution of the ground matrix in which the fibres are embedded. Also, it assumes that the collagen fibres do not support compression, and hence they provide a mechanical contribution only when in tension (this may be taken care of by pre-multiplying each anisotropic term with a Heaviside, or "switching", function).

For the original incompressible HGO model the strain energy Ψ is expressed as a function of one isochoric isotropic deformation invariant (denoted as $\overline{I}_1$) and two isochoric anisotropic invariants (denoted as $\overline{I}_4$ and $\overline{I}_6$). A Lagrange multiplier is used to enforce incompressibility (Holzapfel et al., 2000). Once again it should be stressed that the original HGO model is intended only for the simulation of incompressible materials.

A modification of the original HGO model commonly implemented in finite element codes entails the replacement of the Lagrange multiplier penalty term with an isotropic hydrostatic stress term that depends on a user specified bulk modulus. This modification allows for the relaxation of the incompressibility condition and we therefore refer to this modified formulation as the HGO-C (compressible) model for the remainder of this study.

The HGO-C model has been widely used for the finite element simulation of many anisotropic soft tissues. For example, varying degrees of compressibility have been reported for cartilage in the literature (e.g. Guilak et al., 1995; Smith et al., 2001). It has been modelled as a compressible material using the HGO-C model (e.g. Peña et al., 2007 used Poisson's ratio, $\nu=0.1$ and Pérez del Palomar and Doblaré (2006) used $\nu=0.1$ and $\nu=0.4$). To date, material compressibility of arterial tissue has not been firmly established. Incompressibility was assumed by the authors of the original HGO model and in subsequent studies (e.g. Kiousis et al., 2009). However many studies model arteries as compressible or slightly compressible (e.g. Cardoso et al., 2014, $\nu=0.33$–0.43 and Iannaccone et al., 2014, $\nu=0.475$). In addition to arterial tissue the nucleus pulposus of an intervertebral disc has been modelled as a compressible anisotropic material using the HGO-C model (e.g. Maquer et al., 2014, $\nu=0.475$). Furthermore the HGO-C formulation has been used to simulate growth of anisotropic biological materials, where volume change is an intrinsic part of a bio-mechanical process (e.g. Huang et al., 2012, $\nu=0.3$). However, the enforcement of perfect incompressibility may not be readily achieved in numerical models. As an example, the finite element solver Abaqus/Explicit assigns a default Poisson's ratio of 0.475 to "incompressible" materials in order to achieve a stable solution (Abaqus, 2010) and in this case the HGO-C model must be used (e.g. Conway et al., 2012; Famaey et al., 2012). Despite the widespread use of the HGO-C model, its ability to correctly simulate anisotropic compressible material behaviour has not been established previously:

- The first objective of this study is to demonstrate that the HGO-C formulation does not correctly model an anisotropic compressible hyperelastic material.