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Research Paper

On a new model for inhomogeneous volume growth of elastic bodies

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ABSTRACT

In general, growth characterises the process by which a material increases in size by the addition of mass. In dependence on the prevailing boundary conditions growth occurs in different, often complex ways. However, in this paper we aim to develop a model for biological systems growing in an inhomogeneous manner thereby generating residual stresses even when growth rates and material properties are homogeneous. Consequently, a descriptive example could be a body featuring homogeneous, isotropic material characteristics that grows against a barrier. At the moment when it contacts the barrier inhomogeneous growth takes place. If thereupon the barrier is removed, some types of bodies keep the new shape mainly fixed. As a key idea of the proposed phenomenological approach, we effort the theory of finite plasticity applied to the isochoric part of the Kirchhoff stress tensor as well as an additional condition allowing for plastic changes in the new grown material, only. This allows us to describe *elastic bodies with a fluid-like growth characteristic*. Prominent examples are tumours where the characteristic macro mechanical growth behaviour can be explained based on cellular arguments. Finally, the proposed framework is embedded into the finite element context which allows us to close this study with representative numerical examples.

1. Introduction

Growth phenomena are defined as material increase in size by the addition of mass and are of sociological interest as they appear in various domains of our daily routine: From cells over tissues to entire organisms. Analyses of such phenomena have entered in many different scientific communities, reaching from classical biology and plant biology over medical sciences to engineering and mathematics. The common aim of all these scientific groups is a detailed understanding of growth behaviour for most diverse processes including e.g. plant growth (Srivastava, 2002; Beck, 2010), wound healing (Martin, 1997; Thackham et al., 2008), bone regeneration (Dimitriou et al., 2011; Tal, 2012), tumour growth (Kim et al., 2011), aneurysms growth (Keen and Dobrin, 2000), and in general growth of tissues such as arteries (Helisch and Schaper, 2003), skeletal muscles (Koopman and van Loon, 2009), or heart (Burggren and Keller, 1997; Tomanek and Runyan, 2001), to list only a few obvious examples. However, growth fulfils a variety of biological functions and takes place in three typical forms, namely tip growth, surface growth, and volume growth (BenAmar and Goriely, 2005). In contrast to tip and surface growth, volumetric growth in the bulk of biological systems has been well-documented as it occurs in most systems such as arteries,

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skeletal muscles, heart, airways, and solid tumours (Taber, 1995; Ambrosi and Mollica, 2002; Humphrey, 2003; Cowin, 2004; Moulton and Goriely, 2011; Dervaux and Ben Amar, 2011).

After the perception that soft tissues are highly complex materials, featuring non-linear, anisotropic, and inhomogeneous characteristics, significant endeavours have been directed toward the development of theoretical models for volumetric growth, mostly for soft tissues such as cardiac tissue (Kroon et al., 2009; Göktepe et al., 2010b), tendons (Garikipati et al., 2004), skeletal muscles (Zöllner et al., 2012), tumours (Ambrosi and Mollica, 2002), and vascular tissues (Taber and Humphrey, 2001; Alastrué et al., 2008). However, all these models follow the general statement of growth (Rodriguez et al., 1994) in terms of the multiplicative decomposition of the geometrical deformation gradient as known from elasto-plasticity (Kröner, 1959; Lee, 1969). Basic idea of Rodriguez et al. (1994) is that the deformation gradient is split into a growth part describing the local exchange of mass and a so-called elastic one to ensure compatibility and integrity due to growth. As the deformation gradient is a geometric tensor, the modelling of growth using the multiplicative decomposition is of geometric nature, too. This fact initiated a discussion about the suitability of this approach, see Humphrey and Rajagopal (2002) and Ambrosi et al. (2011). Following Humphrey and Rajagopal (2002), a possible remedy could be the modelling of growth within the mixture theory. Growth could be then described by the evolution of various natural configurations. However, the majority of contributions use the classic multiplicative decomposition of the deformation gradient.

An intensive discussed phenomenon in terms of volumetric growth is the generation of residual stresses inside biological systems (Hoger, 1986; Skalak et al., 1996). Basically, when growth takes place locally, e.g. due to local nutrient supply (Ambrosi and Preziosi, 2012), parts of the body become stretched/compressed to ensure compatibility. These strains are associated with stresses referred to as residual stresses which have been shown to play an important role in the function of soft tissues. The maybe most prominent example in this respect are residual stresses in arteries regulating the stress distribution inside the arterial wall (Rachev and Greenwald, 2003).

In this paper, we aim to study biological systems that grow in an inhomogeneous manner thereby generating residual stresses even when material characteristics are homogeneous. Such growth scenarios are known e.g. from cells (Ateshian and Humphrey, 2012) or tumours (Ambrosi and Preziosi, 2012). However, in order to become more familiar with inhomogeneous growth driven by external constraints, the basic idea is demonstrated on an apple, see Fig. 1. Hereby, a cable tie was attached to the apple during the growth process, see Fig. 1(a). After a growth period of two months the apple was picked and a photo was made directly from this situation, see (b). Alter slicing the cable tie it springs open and by merging both ends a distance ΔL, cf. (c), could be detected. In (d) the bisected apple is illustrated. Two effects can be observed: first, due to the restriction imposed by the cable tie, the apple is prevented to grow homogeneously although its flesh can be characterised to be isotropic and homogeneous (apart from the apple core). This behaviour leads to rank growth especially in the region around the cable tie, cf. (b) and (d). The second effect can be visualised when focusing on the distance ΔL. The burst of the cable tie when cut points out that there are some residual stresses inside the apple. As the mechanical behaviours of the apple and the cable tie are

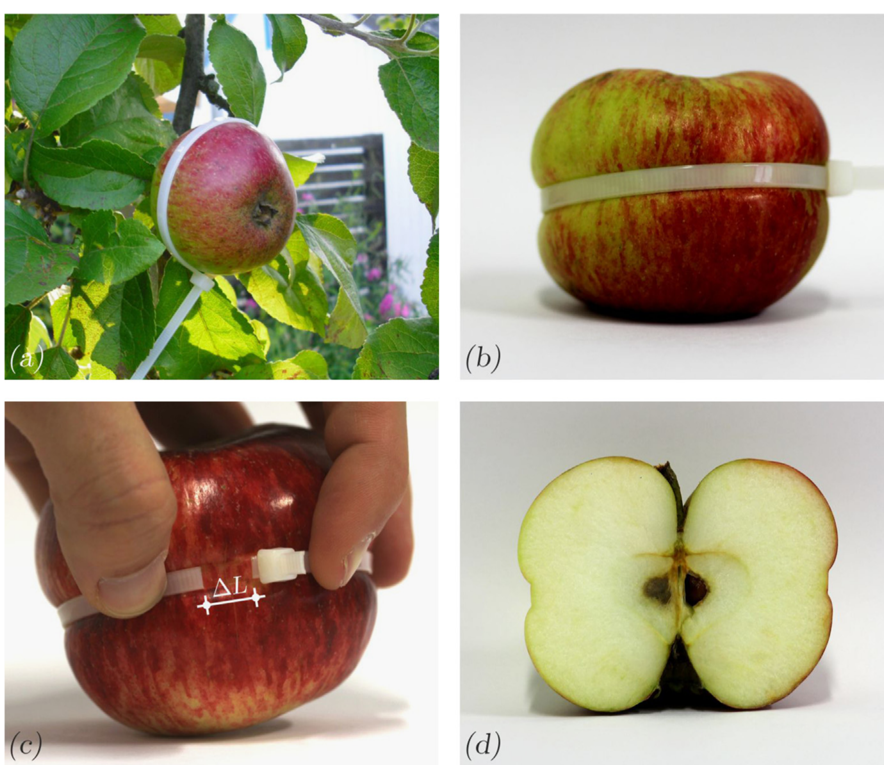

Fig. 1 – Inhomogeneous, locally restricted growth of an apple. (a) Situation on the tree, a cable tie has been loosely attached around an apple. (b) Apple after two months of growth, (c) cut cable tie, and (d) same apple bisected.