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Effects of internal pressure and surface tension on the growth-induced wrinkling of mucosae

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ABSTRACT

Surface wrinkling of mucosae is crucial for the biological functions of many living tissues. In this paper, we investigate the instability of a cylindrical tube consisting of a mucosal layer and a submucosal layer. Our attention is focused on the effects of internal pressure and surface tension on the critical condition and mode number of surface wrinkling induced by tissue growth. It is found that the internal pressure plays a stabilizing role but basically has no effect on the critical mode number. Surface tension also stabilizes the system and reduces the critical mode number of surface patterns. Besides, the thinner the mucosal layer, the more significant the effect of surface tension. This work may help gain insights into the surface wrinkling and morphological evolution of such tubular organs as airways and esophagi.

1. Introduction

The growth and morphogenesis of biological tissues and organs are mediated not only by genetic factors but also by environmental effects, e.g. chemical concentrations and mechanical forces (Taber, 1995; Jones and Chapman, 2012; Li et al., 2012). For example, the early development of solid tumors is distinctly affected by the diffusion of nutrient chemicals within the extracellular matrix. The inhomogeneous distribution of nutrients, caused by such reasons as the consumption of tumors themselves, may engender nonuniform cell proliferation and, consequently, elicit specific structures (e.g., a central necrotic core observed in human cervical carcinoma spheroid) and mechanical stresses (Sutherland, 1988; Tracqui, 2009). It has been believed that these intrinsic stresses incurred by differential volumetric growth closely associate with irregular surface patterns on the tumors and their invasion into host tissues (Dervaux et al., 2011; Pham et al., 2011; MacLaurin et al., 2012). As a matter of fact, growth-induced stresses regulate the morphogenesis of almost all biological things in the realms ranging from plants to animals (Liang and Mahadevan, 2009; Li et al., 2011a, 2011c; Savin et al., 2011; Li et al., 2012).

In the past decades, much effort has been directed towards understanding the formation of surface patterns in a diversity of biological tissues and the underlying physical mechanisms. As a class of typical soft tissues, mucous membranes (or mucosae) exist in the inner surfaces of many living organisms, e.g., airways, arteries, esophagi, stomachs and gastrointestinal tracts. Mucosae grow in the way of volumetric variations and are featured by different surface wrinkles and ridges (Lambert et al., 1994; Taber, 1995; Wiggs

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et al., 1997; Fayed et al., 2010). Such patterns are reminiscent of those induced by buckling in such mechanical systems as a stiff film anchored on a compliant substrate. This kind of composite systems may buckle and evolve into various morphologies when the compressive stress in the film exceeds a critical value (Tanaka et al., 1987; Sultan and Boudaoud, 2008; Breid and Crosby, 2009; Liu et al., 2010). Surface patterns not only play a significant role in many physiological functions of healthy tissues but also are clinically relevant to some diseases (e.g. asthma, inflammation, edema and lymphoma) and, hence, their variations have also been regarded as a pathological phenotype (Wiggs et al., 1997).

Due to its physiological and pathological relevance, surface wrinkling of mucosae has received much attention (Lambert et al., 1994; Hrousis et al., 2002; Yang et al., 2007). Recently, Li et al. (2011b) performed a linear perturbation analysis on the growth-induced wrinkling of esophagi and airways. Moulton and Goriely (2011) studied the circumferential buckling instability of a growing cylindrical tube under an external pressure. These previous studies showed that the wrinkling patterns of mucosae are dictated by geometrical and physical parameters of the system, e.g. the thicknesses and mechanical properties of the mucosal and submucosal layers (Wiggs et al., 1997; Li et al., 2011b). In such tubular organs as airways and esophagi, which have an essential function of transportation, there always exists air or liquid. The intraluminal fluids exert an internal pressure on the inner surface of the organs. In addition, the inner mucosal layer has a pronounced surface tension (Hill et al., 1997; Heil and White, 2002; Heil et al., 2008). Experimental observation in airways suggests that surface tension is closely relevant to the closure and opening of airways (Burger and Macklem, 1968; Heil et al., 2008). Kang and Huang (2010) showed that surface tension may modulate the critical wavelength and the critical swelling ratio at the onset of surface instability induced by water-sucking in a planar hydrogel layer. To date, however, the effects of internal pressure and surface tension on the surface wrinkling of growing cylindrical tubes remain unclear.

In this paper, we will investigate, through combined theoretical analysis and numerical simulations, the effects of internal pressure and surface tension on the stability of airways and esophagi. The critical wrinkling condition and the characteristic mode number of the induced surface pattern in the growing system are explored. This paper is organized as follows. In Section 2, a theoretical model is presented to analyze the growth behavior of mucosae and submucosae with the effects of internal pressure and surface tension. A linear perturbation analysis is performed in Section 3 to predict the critical conditions of wrinkling. Nonlinear finite element simulations are also conducted to verify our analytical solution. The implications and conclusions drawn from the present study are given in Section 4.

2. Model of volumetric growth

2.1. Deformation and stress analysis

Such tubular organs as pulmonary airways and esophagi have a multiple-layered structure, which can be roughly divided into three layers, including an innermost mucosal membrane and a submucosal layer enveloped by a stiff muscular layer (Li et al., 2011b). In this paper, therefore, we consider an isotropic and hyperelastic cylinder containing a mucosal layer and a submucosal layer, which grow either in a fixed tube or without any external constraint. Usually, mucosae are much stiffer than submucosae. Therefore, we assume that the elastic modulus of the mucosa is higher than that of the submucosa. The volumetric growth model originally established by Rodriguez et al. (1994) is employed to analyze growth-induced deformation. In the cylindrical coordinate system, the position of a representative material point at the initial configuration $\mathbf{X}=(R,\Theta,Z)$ transforms to $\mathbf{x}=(r,\theta,z)$ at the current configuration due to tissue growth, as shown in Fig. 1. The bilayer tube has the initial inner radius A, the interface radius B, and the outer radius C. Thus, the initial thicknesses of the mucosal layer and the submucosal layer are $H_m=B-A$ and $H_s=C-B$, respectively. Here and in the sequel, the subscripts m and s denote the quantities defined in the mucosa and submucosa, respectively.

Consider the case of axisymmetric growth, which would lead to axisymmetric deformation, i.e. $r=r(R)$. In the current configuration, the inner, interfacial and outer radii become a, b and c, respectively. The associated deformation gradient tensor is written as $\mathbf{F}=\partial\mathbf{x}/\partial\mathbf{X}=diag(\lambda_1,\lambda_2,\lambda_3)$, where $\lambda_1=\partial r/\partial R$, $\lambda_2=r/R$ and λ_3 are the three principal stretches. Here and in the sequel, the indices 1, 2 and 3 stand for the radial, circumferential, and axial directions, respectively. According to the volumetric growth theory (Rodriguez et al., 1994; Ben Amar and Goriely, 2005), the deformation gradient $\mathbf{F}$ can be decomposed into $\mathbf{F}=\mathbf{A}\cdot\mathbf{G}$, where $\mathbf{G}$ denotes the growth part and $\mathbf{A}$ the elastic deformation part. The growth tensor is assumed as $\mathbf{G}=diag(g_1,g_2,g_3)$, where $g_i(i=1,2,3)$ denote the growth factor in the i-th direction, with $g_i>1$ representing growth and $0<g_i<1$ shrinkage. Assume that the bilayer deforms and grows under the plane-strain conditions, that is, the deformation and growth do not happen in the longitudinal direction. Thus the growth tensor reduces to $\mathbf{G}=diag(g_1,g_2,1)$ and the deformation gradient tensor has the form of $\mathbf{F}=diag(\lambda_1,\lambda_2,1)$. We further assume g_1 and g_2 to be spatially uniform and only consider the isotropic growth, i.e. $g_1=g_2=g>1$. The elastic deformation tensor $\mathbf{A}$ has the form of $\mathbf{A}=diag(\alpha_1,\alpha_2,\alpha_3)$, where the stretch ratios are $\alpha_1=g^{-1}\partial r/\partial R$, $\alpha_2=g^{-1}r/R$ and $\alpha_3=1$. In general, the elastic deformation of living soft tissues yields little volume change. Therefore, the nonlinear responses of mucosae and submucosae can be described by the isotropic and incompressible

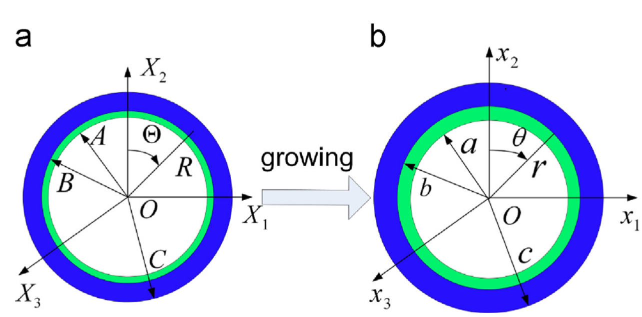

Fig. 1 – A growing bilayer tissue in a cylindrical lumen: (a) initial configuration and (b) current configuration.