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Extremal states of energy of a double-layered thick-walled tube – Application to residually stressed arteries

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ABSTRACT

Various biological tissues are designed to optimally support external loads for complex geometries and mechanobiological structures. This results in complex microstructures of such materials. The design of, for instance, (healthy) arteries, which are in the focus of this work, is characterised by a residually stressed fibre-reinforced multi-layered composite with highly non-linear elastic response. The complex interaction of material properties with the geometry and residual stress effects enables the optimal support under different blood pressures, respectively blood flow, within the vessel. The fibres reinforcing the arterial wall, as well as residual stresses present in the vessel, strongly influence its overall behaviour and performance. Turn-over and remodelling processes of the collagenous fibres occurring in the respective layers – either resulting from natural growth phenomena or from artificially induced changes in loading condition such as stent deployment – support the optimisation of the multi-layered composite structure of arteries for the particular loading conditions present in the artery.

Within this contribution, the overall energetic properties of an artery are discussed by means of the inflation, bending and extension of a double-layered cylindrical tube. Different states of residual stresses and different fibre orientations are considered so that, for instance, representative fibre angles that result in extremal states of the total potential energy can be identified. In view of turn-over and remodelling processes, these orientations are considered to constitute preferred directions of fibre alignment. In summary, the main goal of this work is to calculate optimal material, structural and loading parameters by concepts of energy-minimisation. Several numerical studies show that the obtained values – such as the fibre orientations, the residual axial stretch and the opening angle – are in good agreement with respective physiological parameters reported in the literature.

1. Introduction

The investigation of the mechanical behaviour of soft biological tissues, such as arteries, has gained much attention during the last decades; see e.g. Humphrey (2002) for a general overview. Various biological tissues and structures are designed to optimally support external loading, which often results in complex microstructures of these materials. The design of, for instance, healthy arteries is characterised by a residually stressed fibre-reinforced layer-wise orthotropic composite with highly

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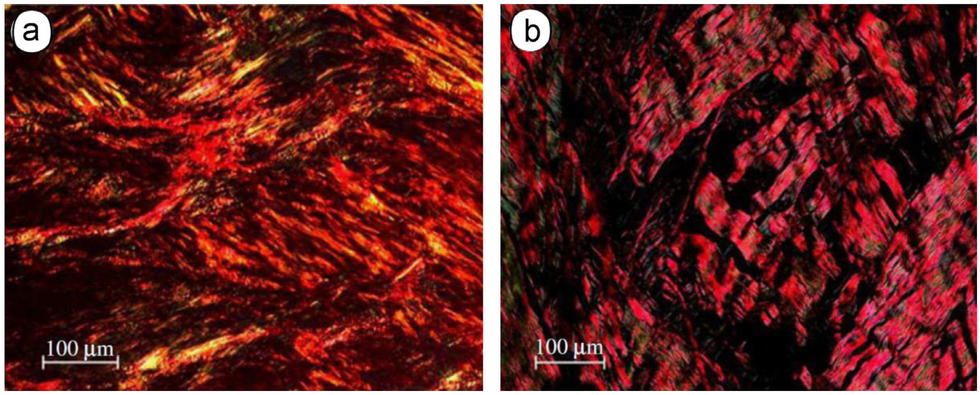

Fig. 1 – Polarised light micrographs of the media (a) and the adventitia (b) of a thoracic aorta wall. The images show two distinct collagen fibre families oriented in the θ–z–plane of the arterial wall. Reprinted from Schriefl et al. (2012), with kind permission from Royal Society Publishing.

nonlinear elastic response. As a result of the interaction between residual stresses and nonlinear anisotropic material properties, arteries may optimally sustain different blood pressures, respectively blood flow, within the vessel. In addition to reducing or rather homogenising the distributions of the respective stress components, the optimal design of the artery e.g. reduces the change in its axial direction under the action of blood flow to a minimum.

Fig. 1 shows two representative micrographs of a thoracic aorta, namely the media (a) and the adventitia (b); the figures are taken from the work by Schriefl et al. (2012). Several advanced constitutive models for the simulation of the mechanical response of arteries have been proposed in the literature and have successfully been applied to realistic patient-specific case studies; see, for instance, the works by Rissland et al. (2009), Holzapfel and Ogden (2010), Balzani et al. (2012) and Creane et al. (2012).

In order to test basic capabilities of the respective constitutive models with application to the simulation of arteries, it is common to analyse an idealised test case, namely the inflation of a thick-walled cylindrical tube. As an advantage, the underlying boundary value problem can be solved (almost) analytically without the need for extensive numerical methods such as the finite element method. Nevertheless, all representative characteristics of the loading conditions and the geometry can be captured; see Holzapfel and Ogden (2003, 2009) and references cited therein. For the present case of application of a residually stressed artery subjected to different blood pressures, commonly combined loading cases including inflation, bending, and axial extension of the thick-walled tube are considered, thereby often neglecting the torsion mode; see the article by Holzapfel et al. (2000) for a particular treatment on arterial wall mechanics. The bending and extension modes are typically related to residual stress states within the arterial wall – which may change in time due to, for instance, ageing effects as discussed in Cardamone et al. (2009) – whereas the inflation refers to loading induced by the blood pressure. Alternative approaches for the inclusion of residual stress states in arteries are discussed in, for instance, Olsson et al. (2006) and Hoger (1985).

The problem of the inflation of a tube is commonly studied by means of hyper-elastic forms together with the underlying equilibrium conditions, respectively the Euler–Lagrange equations; see, for instance, Gent and Rivlin (1952), the monographs by Green and Adkins (1970) as well as Ogden (1997), and the series of papers by Haughton and Ogden (1979, 1980a,b). This allows the investigation of the distribution of stresses under particular loading levels, fibre angles and residual stress states within the multi-layered thick-walled tube; see the contributions by Pipkin and Rivlin (1962) and Spencer et al. (1974), wherein inextensibility of the fibres is assumed. From the mechanical point of view, the fibres reinforce the arterial wall while the residual stresses decrease, e.g. the circumferential stresses within the wall. However, it generally remains unclear which stimulus yields a particular fibre orientation or realignment under changing loading conditions.

Moreover, the interaction of the fibre reorientation with the states of residual stresses shall generally be accounted for; see the work by Alford et al. (2008), wherein the response of an artery in the context of varying properties of the underlying constituents is investigated. Different modelling concepts have been suggested in the literature to simulate the alignment of fibres, commonly denoted as turn-over or remodelling. Particular remodelling formulations are proposed by, for instance, Driessen et al. (2004), where a kinematics-based fibre alignment is suggested, or Humphrey and Rajagopal (2002), wherein a mixture-theory-based remodelling approach is developed. Coaxial states of stresses and strains render the strain energy to take an extremal state, see Sgarra and Vianello (1997), which has motivated alternative remodelling formulation biological tissues with one fibre family, see e.g. Menzel (2005) and Menzel and Waffenschmidt (2009), and two families of fibres, see Menzel (2007). For an overview on different growth and remodelling approaches the reader is referred to the articles by Ambrosi et al. (2011) and Menzel and Kuhl (2012).

Within this contribution, an attempt towards the interpretation of the interactions between loading conditions, states of residual stresses and fibre orientations is made by means of energy-based arguments. As an exemplary constitutive relation, the model proposed by Holzapfel et al. (2000)