SOFTLY SWITCHED ROBUSTLY FEASIBLE MPC FOR CONSTRAINED LINEAR SYSTEMS UNDER SET BOUNDED UNCERTAINTY- LQ-MPC WITH IC ACTION

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Abstract: An efficient plant control under state and output constraints under a wide range of operating conditions is not possible by one controller. Hence, the concept of multiple controllers being switched as required by operating conditions is an attractive option. Quite often, hard switching of the controllers may not be desired due to variety of reasons or not possible at all. The paper considers soft switching of the predictive controllers utilizing the invariant sets theory to constrained linear discrete time invariant system operating under bounded additive disturbances. The applied resulting softly switched predictive controller (SS MPC) with the integral action activates smooth and robustly feasible tracking of the plant output reference trajectory under wide range of operating conditions. The controller performance is illustrated by simulations.

Keywords: predictive control, discrete-time systems, soft switching, invariant sets, feasibility, robust control.

1. INTRODUCTION

Model Predictive Control (MPC) is one of the most popular control techniques thanks to its ability to efficiently handle state and control constrains and deal with existing uncertainty (Kothare *et al.,* 1996; Mayne *et al.,* 2000). In most cases, it is difficult or even impossible to handle full range of the system operating conditions by one universal control strategy. Therefore, arises an idea to utilize several different control strategies and to skilfully switch between them (Brdys *et al.,* 2004; Brdys and Wang, 2005; Grochowski, *et al.,* 2004; Wang *et al.,* 2005). Given a plant to be controlled and a set of objectives to be achieved the model predictive controller is defined by specifying the performance index (function) and constraints on the decision vector and states/outputs of the plant. The constraints and performance index express the control objectives. The MPC as such is then considered as a control strategy determined by the set of control objectives. It is very difficult or even impossible to achieve all the worthwhile operational objectives during plant operation under wide range of the operating conditions. Hence, it is impossible to handle sufficiently well all the operating conditions of the plant by using one control strategy. In order to best adopt the control actions to the actual and predicted conditions, different control strategies were designed. As the operating conditions vary the best control strategy should be selected on line. The overall predictive controller is then the switched controller. Newly developed techniques for smooth switching the control strategies lead to a softly switched MPC (Brdys, *et al.,* 2007; Brdys, *et al.,* 2004; Bryds and Wang, 2005; Grochowski *et al.,* 2004; Wang *et al.,* 2005). This paper proposes a method of soft switching the MPC controllers taking full advantage of Integral Control methodology in order to reject constant and/or slowly varying disturbances. Moreover, feasibility of controllers operating on their own and during the switching phase is guaranteed by utilizing the properties of invariant sets (Rossiter, 2006; Wang, *et al.,* 2005).

The paper is organised as follows. Section 2 formulates the control problem and presents the basic definitions. General information about determining the invariant sets as well as the way of its utilizing into control process are presented in Section 2. Section 3 derives the soft switching algorithm and investigates the properties of resulting Softly Switched robustly feasible model predictive controller. Finally, the derived controller is tested based on a simple second order dynamic system and the simulation results are presented in Section 4.

2. CONTROL STRATEGY

This paper considers discrete state space model with additive uncertainty as:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k \\ y_k &= Ex_k \end{aligned} \tag{1}$$

where: w_k is the disturbance input which values belong to a bounded set W.

Employed predictive controller with linear quadratic cost function (LQ MPC) is extended by an integral action what has enabled for minimizing the output tracking error (Deinrych, *et al.,* 2007). Such an approach for the system (1) leads to formulation:

$$\underbrace{\begin{bmatrix} x_{k+1} \\ x^{j}_{k+1} \end{bmatrix}}_{\bar{x}_{k+1}} = \underbrace{\begin{bmatrix} A & 0 \\ E & I \end{bmatrix}}_{\bar{A}} \underbrace{\begin{bmatrix} x_k \\ x^{j}_{k} \end{bmatrix}}_{\bar{x}_k} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{\bar{B}} u_k + \underbrace{\begin{bmatrix} w_k \\ y_{ref} \end{bmatrix}}_{r} \quad (2)$$

$$y_{k+1} = \underbrace{\begin{bmatrix} E & 0 \end{bmatrix}}_{\bar{E}} \begin{bmatrix} x_k \\ x^{i}_{k} \end{bmatrix}$$

Control law is based on the feedback control resulting from solving the optimisation problem with the linear-quadratic cost function (Pluymers, 2005, Rossiter, 2003):

$$J(x_k) = \min_{u_{0|k},\dots,u} \left\{ \sum_{i=0} \left(\hat{x}_{i|k} \, Q \, \hat{x}_{i|k} + \hat{u}_{i|k} \, R \, \hat{u}_{i|k} \right) \right\} \quad (3)$$

with respect to constraints:

$$\hat{x}_{i|k} \quad X, i \quad \{0,\dots, \quad \} \quad (4)$$

$$\hat{u}_{i|k} \quad U, i \quad \{0,\dots, \quad \} \quad (5)$$

where: the X, U are compact polyhedral sets. This approach is called the dual mode MPC. As a consequence, the control law is as follows:

$$u_i = \quad Kx_i + c_i, i \quad \{0,\dots,H_u \quad 1\} \quad (6)$$

$$u_i = \quad Kx_i, i \quad \{H_u,\dots, \quad \} \quad (7)$$

where the parameter K is the result of solving the cost function (3) on infinite control horizon (second mode of the dual mode MPC) and the degrees of freedom c are the result of solving the quadratic problem on the finite horizon to deal with constraints (4-5). For optimal K (3) takes form:

$$J_k(x_k) = \min_{c} \sum_{i=0}^{Hu \quad 1} c_i^T S_{Dc} c_i \quad (8)$$

where S_{Dc} is a solution of Lyapunov equation (Rossiter, 2003; Deinrych, *et al.,* 2007):

$$\hat{Q} = B \quad B + R, \qquad = \quad + \quad Q \quad KRK \quad (9)$$

Feasibility of the controller is guaranteed by utilizing the invariant sets as the extra constraints in the predictive optimisation procedure (Deinrych, *et. al.,* 2007; Kerrigan, 2000; Kerrigan and Maciejowski, 2004). Two kinds of invariant sets are utilized to do it (Gilbert and Tin Tan, 1991; Pluymers 2005):

- the MAS (Maximal Admissible Set) defined for system based on linear control law (7);
- the MCAS (Maximal Control Admissible Set) based on the control law (6) with active d.o.f. parameters.

The MAS presents the set of states for which the following equations hold:

$$Mx_0 \quad d \quad Mref \quad (10)$$

where matrices M, d and $Mref$ are the result of system expansion in order to satisfy constraints (10) (Deinrych, *et al.,* 2007; Szumski and Szczygielski, 2006).

The MCAS is the set of states for which optimisation problem of considered control strategy is feasible. The constraints (4-5) are fulfilled thanks to using a d.o.f. on finite control horizon (Rossiter, 2003; Deinrych, *et al.,* 2007):

$$Mx_0 + Nc + Mref \quad d \quad (11)$$

The matrices M, N, $Mref$ and d are the result of system expansion. The usage of invariant sets of the above form guarantees achieving optimisation problem feasibility in deterministic case. In order to guarantee the robust feasibility under presence of uncertainty the invariant sets have to be replaced by their robust versions. One of the ways of designing robust invariant sets is adding an extra constraints during the determination and computation of theses sets taking into account existing bounded uncertainty for instance by utilization of the Pontriagin difference. Plant states at the end of the control horizon must belong to robust MAS in order to ensure control feasibility (Kerrigan, 2000; Deinrych, *et al.,* 2007; Pluymers, *et al,* 2005).

A single control strategy for system (1) is represented by: state and control constraints (X, U), reference signals (y_{ref}), weighting matrices in the cost function (Q,R) and control horizon (H_u). Therefore, the single control strategy can be formulated as follows:

$$J^{old}(x_k) = \min_{u_{0|k},\dots,u} \left\{ \sum_{i=0} \left(\hat{x}_{i|k} \, Q^{old} \hat{x}_{i|k} + \hat{u}_{i|k} \, R^{old} \hat{u}_{i|k} \right) \right\} \quad (12)$$

$$\hat{x}_{i|k} \quad X^{old}, i \quad \{0,\dots, \quad \} \quad (13)$$

$$\hat{u}_{i|k} \quad U^{old}, i \quad \{0,\dots, \quad \} \quad (14)$$

$$y_{ref} = y^{old}_{ref} \quad (15)$$

$$\hat{w}_{i|k} \quad W^{old}, i \quad \{0,\dots, \quad \} \quad (16)$$

$$\hat{x}_{H_u|k} \quad MAS^{old}_{r} \sim W^{old} \quad (17)$$

$X^{old} \quad X$, $U^{old} \quad U$ are compact polyhedral sets. Unfortunately, the control strategy described above does not guarantee an efficient plant control under wide range of possible operating conditions (Brdys, *et al.,* 2007; Brdys, *et al.,* 2004; Grochowski, *et al.,* 2004). For a particular plant number of control strategies can be distinguished. The desired (new) control strategy is described as follows:

$$J^{new}(x_k) = \min_{u_{0|k},\dots,u} \left\{ \sum_{i=0} \left(\hat{x}_{i|k} \, Q^{new} \hat{x}_{i|k} + \hat{u}_{i|k} \, R^{new} \hat{u}_{i|k} \right) \right\} \quad (18)$$

$$\hat{x}_{i|k} \quad X^{new}, i \quad \{0,\dots, \quad \} \quad (19)$$

$$\hat{u}_{i|k} \quad U^{new}, i \quad \{0,\dots, \quad \} \quad (20)$$

$$y_{ref} = y^{new}_{ref} \quad (21)$$

$$\hat{w}_{i|k} \quad W^{new}, i \quad \{0,\dots, \quad \} \quad (22)$$

$$\hat{x}_{H_u|k} \quad MAS^{new}_{r} \sim W^{new} \quad (23)$$

The switching moments, switching time and switching method are the most important parameters to be tuned in the soft switching mechanism, however they are not investigated in this paper. Paper focuses on designing the switching algorithm that