PARALLEL COMPENSATOR FOR CONTROL OF MULTIVARIABLE SYSTEMS WITH DIFFICULT PLANTS

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Abstract: Parallel compensator applied for control of SISO and MIMO plants is proposed. It is based on the replacement plant (RP) which connected in parallel to the plant changes the property of the latter. There is some freedom in choosing the transfer function (TF) of the RP. It is shown that the numerator polynomial of the RPTF determines the characteristic equation of the closed loop (CL) system. Creating this polynomial from some admissible fast modes assures fast transients of the CL system. For MIMO plants the elements of the diagonal RPTF are designed separately, which simplifies the process of design.

Keywords: Parallel compensator, linear systems, multivariable systems, high gain.

1. INTRODUCTION

The plants with pure time delay, or nonminimum phase and/or of higher order belong to the so called difficult plants, for which it is difficult to design a regulator assuring some appropriate accuracy of the control. For these plants an insignificant increase of the regulator gain causes instability and for small gain the closed loop system has unsatisfactory accuracy in steady state or slow transients.

For the plants with pure time delay Smith (1958) proposed a compensator which effectively takes the delay outside the loop and allows a feedback design based on the plant dynamics without delay.

The idea of the parallel compensator for the systems with nonminimum phase plants was introduced in (Gessing, 2004), by the author of the present paper. The method of the choice of the replacement plant was described there with taking mainly into account the accuracy in steady state.

Another approach to parallel compensator was presented in (Deng *et al.*, 1999) and in its references, where the plants with structured uncertainty were considered. In comparison to (Gessing, 2004) and the present paper the considerations of (Deng *et al.*, 1999) are significantly more complicated, though they concern only minimum phase plants.

Above remarks concern the systems with SISO plants. In the present paper, using the idea of (Gessing, 2004) a new method of design of the replacement plant (RP) both for SISO and MIMO plants is described. The presented method makes it possible to design the systems with faster (shorter) transients. In both the cases of SISO and MIMO plants the speed of the transients depends on the numerator polynomials of the RPTF-s, which may be in some degree of freedom freely chosen. For MIMO plants the applied decoupling simplifies the process of design.

2. PARALLEL COMPENSATOR

In this section the idea of the parallel compensator introduced in (Gessing, 2004) will be reminded.

Consider the linear plant described by the transfer function (TF)

$$G(s) = \frac{Y(s)}{U(s)} = \frac{L(s)}{M(s)} \tag{1}$$

where $Y(s)$ and $U(s)$ are the Laplace transforms of the plant output and input, respectively, while $L(s)$ and $M(s)$ are polynomials of m-th and n-th degree, respectively, $m < n$. Assume that the plant is stable, that is its poles $p_i,\ i = 1, 2, ..., n$ have negative real parts i.e. $Rep_i < 0$.

The parallel compensator is described by the TF

$$G^p(s) = \frac{Y^p(s)}{U(s)} = G^r(s) - G(s) \tag{2}$$

and its idea, as it was noted in (Gessing, 2004) is similar to that of the Smith predictor. Here $Y^p(s)$ is the Laplace transform of the output y^p of the compensator, while $G^r(s)$ is the TF which will be appropriately chosen.

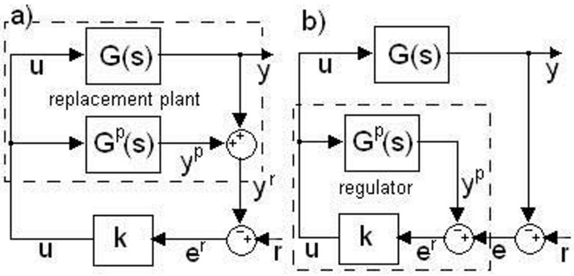

Fig. 1. The equivalent block diagrams of the system with parallel compensator.

Note that in the proposed structure shown in Fig. 1a the TF $G^r(s)$ of the replacement plant outlined by the dashed line is described by

$$\frac{Y^r(s)}{U(s)} = G(s) + G^p(s) = G^r(s) \tag{3}$$

Of course, to implement a closed loop (CL) stable system with the reference signal r determining the demanded output y the TF $G^r(s)$ should fulfill some demands.

In the case of regulation when $r = const$ the error in a constant steady state is mainly interesting, therefore for some constant steady state values it should be

$$y^p = 0, \quad y^r = y, \quad e^r = r - y^r = r - y \tag{4}$$

The latter condition will be fulfilled if

$$G^r(0) = G(0) \tag{5}$$

3. APPROXIMATE DESCRIPTION OF THE CL SYSTEM

The equivalent block diagram of the system from Fig. 1a is shown in Fig. 1b. Note that the part of the system outlined by the dashed line contains the elements of the regulator based on the parallel compensator. Assuming that the CL system is stable and has appropriate phase margin under high gain k, the regulator in the system is described by the following TF $C(s)$

$$C(s) = \frac{U(s)}{E(s)} = \frac{k}{1 + kG^p(s)} \approx \frac{1}{G^p(s)} \tag{6}$$

Note that if the TF $G^r(s)$ has the relative degree equal to one, then the TF (3) of the regulator has the degree of numerator polynomial greater by one from that of the denominator. Thus the regulator (6) has the derivative part.

Accounting (6) we obtain the following formula describing the CL system

$$\frac{Y(s)}{R(s)} = \frac{G(s)/G^p(s)}{1+G(s)/G^p(s)} = \frac{G(s)}{G^p(s)+G(s)} = \frac{G(s)}{G^r(s)} \tag{7}$$

The formula (7) may be used for designing the TF $G^r(s)$. One such a possibility will be discussed the next section.

4. DESIGN OF THE REPLACEMENT PLANT TRANSFER FUNCTION

Denote by

$$G^r(s) = \frac{L^r(s)}{M^r(s)} \tag{8}$$

a stable replacement plant (3) with minimum phase zeros. Thus the polynomials $L^r(s)$ and $M^r(s)$ are Hurwitz polynomials. One way of designing $G^r(s)$ is to choose

$$M^r(s) = M(s) \tag{9}$$

$$L^r(s) = l(1 + sT)^{n-1}, \quad l = L(0) \tag{10}$$

so the condition (5) is fulfilled.

Denote by $\varphi^r(\omega)$ the phase of the frequency response $G^r(j\omega) = L^r(j\omega)/M(j\omega)$. Let the phase $\varphi^r(\omega)$ fulfills the inequality

$$\max_{0 \leq \omega < \infty} |\varphi^r(\omega)| < \varphi \tag{11}$$

where φ is a given positive value of the phase. It may be for instance $\varphi = 90°$, then $G^r(s)$ is strictly positive real (SPR) TF for which $ReG^r(j\omega > 0)$. For our goal it may be the weaker demand: $\varphi = 130°$. The later determines the absolute value of the phase of the vector on the Nyquist plane attached in the origin, which is tangent to the so called M_c circle for $M_c = 1.3$. The circle $M_c = 1.3$ determines the locus of the points on the Nyquist plane for which the maximal absolute value of

$$M_c = \max_{0 \leq \omega < \infty} \left| \frac{kG^r(j\omega)}{1 + kG^r(j\omega)} \right| \tag{12}$$

is equal to 1.3. The value $M_c = 1.3$ is used as the recommended value of a magnitude during