DESIGNING NONLINEAR CONTROL SYSTEMS BY STATE-SPACE FLOW GRAPH OPTIMIZATION

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Abstract: This paper presents a method for planning optimal control for nonlinear processes by using discrete optimization algorithms. The principal concept is to analyze the autonomous dynamics and to use its properties in the design of an optimal state-space trajectory. The analysis is performed in a multi-stage procedure, based on a proper decomposition of an operational subspace into a set of segments. The arrangement of these segments is transformed into a flow graph structure. Flow values characterize the cost of driving the operational point between two adjacent nodes. A discrete optimization algorithm is used to search the state-space graph for an optimal path.

Keywords: nonlinear control systems, control system design, optimization, numerical methods

1. INTRODUCTION

There are two main groups of methods that can be used for solving nonlinear control problems for dynamical systems. Classical control analytical methods, which rely on finding a local (or global) minimum of an assumed control cost function. Known analytical tools like the calculus of variation, the Euler-Lagrange equation and the Hamilton-Pontryagin criterion certain necessary conditions. There is a sufficient condition for the existence of a minimum (Lewis 1992) resulting from the Bellman's approach. However, the methods mentioned above cannot be directly used in the case of dynamical processes that are described by the models which contain hard nonlinearities (the lack of derivatives of functions in some points of its domain). The same restriction concerns the cost function. Another disadvantage of classical methods is the complexity of the necessary symbolic transformations.

There are many numerical techniques approaching the optimal control trajectory by iteratively computing solutions of nonlinear two-point boundary-value problems (Kirk 1970). However, these algorithms (steepest descent, variation of extremes or quasilinearization) require process description by means of differentiable functions without constrains imposed on control and state variables. Another bunch of numerical procedures can be applied when differential equations are approximated by difference equations. After discretization some effective nonlinear programming algorithms (gradient or nongradient) can be applied, such as the conjugate direction methods (Nash 1996), (Bertsekas 2000). The main disadvantage of these methods lies in converging to local minima and in the assumption that the feasibility region of solution is continuous, whereas the control signals in practical problems can be for instance limited to certain ON-OFF positions. Discrete dynamics programming (Bertsekas 2005) avoids the difficulties mentioned above, but falls into the curse of dimensionality, characteristic for the Bellman's approach leading to drastic increase of the computation time. This paper proposes an alternative technique for finding solution of the following minimum control-effort problem. Let us consider a nonlinear system of an n-th order having m controls described by the state equations

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) \tag{1}$$

where $\mathbf{f}$ is a vector function defining the system dynamics, $\mathbf{x} \in \mathcal{R}^n$ is a state, $\mathbf{u} \in \mathcal{R}^m$ is a control signal and t means time. The variables $\mathbf{x}$ and $\mathbf{u}$ can be constrained to admissible regions $\mathcal{X}$ and $\mathcal{U}$, respectively: the set $\mathcal{X} = P \backslash Z$, where P is a limited operational subspace of $\mathcal{R}^n$ and Z stands for a forbidden zone.

Our objective is to find the optimal control $\mathbf{u}^*(t) \in \mathcal{U}$ along with the corresponding state trajectory $E^* = E(\mathbf{u}^*(t)) \in \mathcal{X}$, which transits the dynamical system (1) from its initial state $\mathbf{x}(0) = \mathbf{x}_0$ to a specified target state $\mathbf{x}(T) = \mathbf{x}_k$ and minimizes the cost functional

$$J(E(u)) = \int_0^T (\sum_{i=1}^{m} \beta_i |u_i(t)|) dt \tag{2}$$

where β_i, $i = 1, \ldots, m$ are nonnegative weights factors and T is transmition time resulting from the optimal control procedure.

Our main objective is to present a formal description of the proposed algorithm along with two simple applications.

2. FORMAL DESCRIPTION

We start the formal description by separating *autonomous* and *forced* components in the dynamic model (1) of the analyzed process:

$$\dot{\mathbf{x}} = \dot{\mathbf{x}}_x + \dot{\mathbf{x}}_u = \mathbf{f}_x(\mathbf{x}, t) + \mathbf{f}_u(\mathbf{x}, \mathbf{u}, t) \tag{3}$$

where $\mathbf{f}_u(\mathbf{x}, 0, t) = 0$ and
$\mathbf{x}_x$,$\mathbf{x}_u$ are two fictitious state "contributors",
$\dot{\mathbf{x}}_x = \mathbf{f}_x(\mathbf{x}, t)$ represents *autonomous dynamics*,
$\dot{\mathbf{x}}_u = \mathbf{f}_u(\mathbf{x}, \mathbf{u}, t)$ portrays *forced dynamics*.

The above decomposition makes a principal stage of the following analysis of the process autonomous dynamics. The main idea of the presented method consists in an apt utilization of the autonomous dynamic's properties in finding the optimal control strategy in terms of (2).

Let us introduce some terminology necessary for further development.

Definition 1. **System's trajectory $E(\mathbf{u})$**
Any sequence of states defined in the state-space of a given dynamical system with a feasible control input set $\mathcal{U}$ is said to be a system's trajectory or a trajectory of operational points of the space.

Definition 2. **Operational subspace P**
A bounded subset P of the state-space in the form of a hypercube in $\mathcal{R}^n$, which is taken into account while seeking an optimal trajectory is said to be an operational subspace.

Definition 3. **Forbidden zone Z**
A subset Z of P prohibited for operational points is referred to as a forbidden zone. This means that the sought optimal trajectory cannot enter it.

Definition 4. **Transition vector Λ**
A transition vector Λ is an ordered set of two elements $\{x_0, x_k\}$

$$\Lambda = \{(x_0, x_k) : x_0, x_k \in E(\mathbf{u})\} \tag{4}$$

where x_0 is the first element and x_k is the last element of the sought optimal trajectory.

Definition 5. **Cost function $J(E(\mathbf{u}))$ of a trajectory and control**
Let $\mathcal{A}$ be a set of all trajectories belonging to a given operational subspace P. A function of the class $\mathcal{A} \rightarrow \mathcal{R}$ which assigns a real value to each trajectory $E \in \mathcal{A}$ is said to be a cost function $J(E(\mathbf{u}))$ of this trajectory.

Definition 6. **Optimal trajectory E^***
Let $\Xi \subset \mathcal{A}$ be the subset of all possible trajectories which start at a given point x_0 and terminate at x_k. We say that a trajectory E^* is optimal if it satisfies the following conditions:

$$\forall_{E \in \Xi} J(E) \geq J(E^*) \tag{5}$$

and

$$\forall_{\mathbf{x} \in E^*} \mathbf{x} \in P \setminus Z \tag{6}$$

Definition 7. **Segmentation of an operational subspace**
Segmentation of a given operational subspace P into a set of N_s segments is defined as follows:

$$P = \bigcup_{j=1}^{N_s} \Phi_j \tag{7}$$