# Pursuit of an Evader with Hybrid Dynamics ⋆

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**Abstract:** A pursuit-evasion differential game with bounded controls and prescribed duration is considered. The evader has a finite number of possible dynamics, while the dynamics of the pursuer is fixed. The evader can change its dynamics several times during the game. The pursuer knows all possible evader dynamics, but not the actual one. The optimal pursuer feedback strategy in this game is obtained. This strategy is robust with respect to the order of the evader dynamics during the game, as well as instants of changing the dynamics. For this strategy, the capture zone is constructed. An illustrative example is presented.

*Keywords:* Pursuit-evasion game, differential game, hybrid dynamics, robust feedback strategy, capture zone.

## 1. INTRODUCTION

During several recent decades, control problems for hybrid dynamics systems have attracted an attention of many researchers. In the open literature, devoted to this topic, mostly control problems with a single decision maker are studied (see e.g. Riedinger et al. (2003); Sussmann (1999); Utkin (1983) and references therein), while control problems with two and more decision makers (games with hybrid dynamics) are investigated much less. In Grigorenko (1991), a differential game of pursuit of a single evader by a group of pursuers is considered. The structure of the game dynamics is changed by the evader once during the game. Sufficient conditions for the existence of the game solution are obtained. In Mitchell et al. (2005), the reachability sets for pursuit-evasion games with nonlinear hybrid dynamics are numerically constructed by using solutions of time-dependent Hamilton-Jacobi equations. In Gao et al. (2007), a general pursuit-evasion differential game with hybrid dynamics is studied by using the viability theory and non-smooth analysis.

The pursuit-evasion game, considered in this paper, is a mathematical model of an interception engagement between two moving vehicles, an interceptor $P$ (*pursuer*) and a target $E$ (*evader*). The dynamics of each vehicle is approximated by a first-order transfer function with time constants $\tau_p$ and $\tau_e$, respectively. Moreover, it is assumed that the lateral acceleration commands of the pursuer and the evader are bounded by the constants $a_p^{\max}$ and $a_e^{\max}$, respectively. Thus, the dynamics of each player is completely described by the respective vector $\omega_i = (a_i^{\max}, \tau_i)$, $i = p, e$. The cost function of the game is the distance of closest approach (miss distance). The fixed dynamics version of this game (prescribed vectors $\omega_p$ and $\omega_e$) has been studied extensively in the open literature, see Shinar (1981); Shima and Shinar (2002); Gutman (2006). It was shown that this game has a saddle-point solution in feedback strategies. The solution leads to the decomposition of the game space into two regions (singular and regular) of different optimal strategies. The game space decomposition is completely determined by the pair $(\omega_p, \omega_e)$. This pair also determines the existence or non-existence of a capture zone - the set of all initial positions of the game for which the game value equals zero.

In Shinar et al. (2007), the pursuit of an evader with fixed dynamics by a pursuer with hybrid dynamics is studied, while in Shinar et al. (2009), the evasion from a pursuer with fixed dynamics by an evader with hybrid dynamics is analyzed. In these papers, it was established that the optimal order of dynamics for the player with hybrid dynamics is: from a smaller time constant to a larger one in such a way that the player's agility decreases. Moreover, in each case, the optimal switch moment was derived in a closed form. It is important to note that these moments depend only upon the dynamic modes of the hybrid player. The capture zone of the hybrid pursuer and the escape zone of the hybrid evader also were constructed.

In the present paper, the case of fixed pursuer and hybrid evader is treated but from the viewpoint of a pursuer. It is assumed that the vector $\omega_p$ is fixed, while the vector $\omega_e$ belongs to a given set $\Omega_e \triangleq \{\omega_e^1,\ \omega_e^2,\ ..., \omega_e^N\}$, switching from one value to another any number of times during the

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game. The choice of the evader dynamics and the switch moments can be considered as additional elements of the evader control, not known to the pursuer. Therefore, a guaranteeing pursuer strategy has to be robust not only with respect to the evader acceleration command, but also with respect to the order of the evader dynamics during the game, as well as the instants of changing the dynamics.

## 2. PROBLEM FORMULATION

### *2.1 Engagement Model*

A planar engagement between two moving objects - an interceptor (*pursuer*) and a target (*evader*) - is considered. The schematic view of this engagement is shown in Fig. 1. The $X$ axis of the coordinate system is aligned with the initial line of sight. The origin is collocated with the initial pursuer position. The points $(x_p, y_p)$, $(x_e, y_e)$ are the current coordinates; $V_p$ and $V_e$ are the velocities and $a_p$, $a_e$ are the lateral accelerations of the pursuer and the evader respectively; $\varphi_p, \varphi_e$ are the respective angles between the velocity vectors and the reference line of sight; and $y = y_e - y_p$ is the relative separation normal to the initial line of sight.

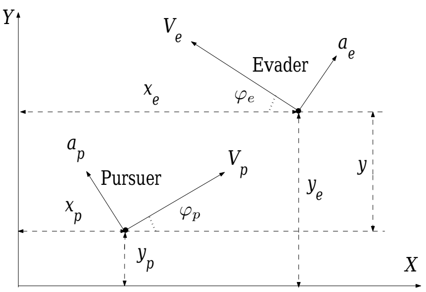


Fig. 1. Interception geometry

It is assumed that the dynamics of each object is expressed by a first-order transfer function with the time constants $\tau_p$ and $\tau_e$, respectively. The velocities and the bounds of the lateral acceleration commands of both objects are constant.

If the aspect angles $\varphi_p$ and $\varphi_e$ are small during the engagement then the linearized engagement model is (Shinar (1981))

$$\dot{x} = Ax + bu + cv, \quad x(0) = x_0, \tag{1}$$

where the state vector is $x = (x_1, x_2, x_3, x_4)^T = (y, \dot{y}, a_e, a_p)^T$, the superscript $T$ denotes the transposition,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1/\tau_e & 0 \\ 0 & 0 & 0 & -1/\tau_p \end{bmatrix}, \tag{2}$$

$$b = (0, 0, 0, a_p^{\max}/\tau_p)^T, \quad c = (0, 0, a_e^{\max}/\tau_e, 0)^T, \tag{3}$$

$$x_0 = (0, V_e \sin\varphi_e(0) - V_p \sin\varphi_p(0), 0, 0)^T. \tag{4}$$

The normalized lateral acceleration commands of the evader $v(t)$ and the pursuer $u(t)$ satisfy the constraints

$$|v(t)| \le 1, \quad |u(t)| \le 1, \quad 0 \le t \le t_f. \tag{5}$$

It is supposed that the functions $v(t)$, $u(t)$ are measurable on $[0, t_f]$ and are bounded according to (5).

**Remark 1.** The angles $\varphi_p$ and $\varphi_e$ remain small in short duration end-game with high velocities. This fact was demonstrated by simulation (see e.g. Fig. 5 in Turetsky and Glizer (2007)). The corresponding linearization is widely used in the literature.

**Remark 2.** The engagement model (1)-(3) is completely determined by two vectors $\omega_e = (a_e^{\max}, \tau_e)$ and $\omega_p = (a_p^{\max}, \tau_p)$ called in the sequel the *dynamic modes* of the evader and pursuer, respectively.

### *2.2 Hybrid Dynamics Game*

It is assumed that the mode $\omega_p$ in (1) – (3) is fixed, while the evader has hybrid dynamics: the mode $\omega_e$ is chosen by the evader a finite number of times during the engagement from the prescribed $N$-element set

$$\Omega_e = \{\omega_e^1,\ \omega_e^2,\ ..., \omega_e^N\}, \quad \omega_e^i \ne \omega_e^j, \quad i \ne j, \quad N \ge 1, \tag{6}$$

where $\omega_e^i = (a_{e,i}^{\max}, \tau_{e,i})$, $i = 1, ..., N$.

Consider the game with the dynamics (1), constraints (5) and the performance index

$$J = |x_1(t_f)|. \tag{7}$$

Note that (7) is the miss distance. The objective of the pursuer is minimizing (7) and of the evader it is maximizing (7), by means of feedback strategies $u(t, x)$ and $v(t, x)$, respectively.

It is assumed that the vector $\omega_p$, the set $\Omega_e$ and a current engagement position $x(t)$ are known to both players. However, the pursuer has no information on a current evader dynamic mode $\omega_e$. This game is called the Original Hybrid Dynamics Game (OHDG). In this paper, the optimal pursuer behavior in the OHDG is analyzed.

## 3. FIXED DYNAMICS GAME

If $N = 1$, the OHDG becomes a game with fixed dynamics of both players. In the sequel, this game is called the Original Fixed Dynamics Game (OFDG). It was solved in Shinar (1981).

### *3.1 Zero-Effort Miss Distance*

The solution of the OFDG is based on its scalarization by introducing a new state variable

$$Z(t) = Z(t; \omega_e, \omega_p) = d^T \Phi(t_f, t; \tau_e, \tau_p) x(t; \omega_e, \omega_p), \tag{8}$$

where $x(t; \omega_e, \omega_p)$ is the state vector of (1), $\Phi(t_f, t; \tau_e, \tau_p)$ is the transition matrix of the homogeneous system $\dot{x} = Ax$, $d^T = (1, 0, 0, 0)$. The value of the function $Z(t)$ has the following physical interpretation. If $u \equiv 0$ and $v \equiv 0$ on the interval $[t, t_f]$, then the miss distance $|x_1(t_f)|$ equals $|Z(t)|$. Therefore, this function is called the zero-effort miss distance (ZEM). It is given explicitly by

$$Z(t) = x_1(t) + (t_f - t)x_2(t) + \tau_e^2 \Psi\left((t_f - t)/\tau_e\right) x_3(t) - \tau_p^2 \Psi\left((t_f - t)/\tau_p\right) x_4(t), \tag{9}$$

where $\Psi(\xi) \triangleq \exp(-\xi) + \xi - 1 > 0$, $\xi > 0$. By introducing a new independent variable (time-to-go) $\vartheta = t_f - t$ and using (9), it can be shown that the function of $\vartheta$

$$\tilde{Z}(\vartheta) = \tilde{Z}(\vartheta; \omega_e, \omega_p) \triangleq Z(t_f - \vartheta; \omega_e, \omega_p) \tag{10}$$