Ain Shams Engineering Journal (2016) xxx, xxx–xxx

Ain Shams University

Ain Shams Engineering Journal

www.elsevier.com/locate/asej
www.sciencedirect.com

ENGINEERING PHYSICS AND MATHEMATICS

Thermal convection in a Kuvshiniski viscoelastic nanofluid saturated porous layer

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Received 29 August 2015; revised 30 October 2015; accepted 29 November 2015

KEYWORDS

Kuvshiniski viscoelastic fluid;
Thermal instability;
Porous medium;
Nanofluid;
Brownian motion;
Thermophoresis

Abstract The thermal convection in a horizontal layer of a porous medium saturated with a viscoelastic nanofluid was studied analytically. A Kuvshiniski-type constitutive equation is used to describe the behavior of viscoelastic nanofluids. The model used for the viscoelastic nanofluid incorporates the effects of Brownian motion and thermophoresis. A physically more realistic boundary condition than the previous ones on the nanoparticle volume fraction is considered i.e. the nanoparticle flux is assumed to be zero rather than prescribing the nanoparticle volume fraction on the boundaries. Using linear stability theory, the exact analytical expression for the Darcy–Rayleigh number is obtained in terms of various non-dimensional parameters. Results indicate that the coefficient of viscosity, porous medium and nanoparticles significantly influences the stability characteristics of the system. The effect of various parameters on the thermal instability is also presented graphically.

1. Introduction

In current year, huge amount of research and development works have been devoted to nanotechnology. According to Nano Science and Foundation (NSF) the market of nanotechnology will exceed $1 trillion in the USA alone by 2015. About a decade ago Argonne Laboratory started to develop special type of colloid suspension of nanopowders with diameter ranging from 1 nm to 100 nm in host fluids such as water, oil or Ethylene Glycol. Fluids with nanoparticles suspended in them are called nanofluids, a term proposed by Choi [1]. Nanofluids can be considered the next-generation heat transfer fluids as they offer exciting new possibilities to enhance heat transfer performance compared to pure fluids [2]. In recent years, nanofluids are used in automotive radiators, lubrication, additives for fuels, shock absorbers replacing or along with the traditional materials used for similar purposes [3]. The recent articles by Yu and Xie [4], Aybar et al. [5], Sharma et al. [6] and Sheikholeslami et al. [7–10] have covered the latest developments in this field in detail.

A comprehensive survey of convective transport in nanofluids was made by Buongiorno [11]. He noted that the

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Peer review under responsibility of Ain Shams University.

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http://dx.doi.org/10.1016/j.asej.2015.11.023
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Nomenclature

a	dimensionless wave number	μ	viscosity
a_c	critical wave number	$\bar{\mu}$	effective viscosity
C	specific heat	ρ	density of the nanofluid
D	diameter of nanoparticles	λ	coefficient of viscoelasticity
D_B	Brownian diffusion coefficient	ρ_0	reference density of nanofluid
D_T	thermophoretic diffusion coefficient	ρ_p	density of nanoparticles
$\vec{\mathbf{g}}$	acceleration due to gravity	(ρc)	heat capacity
k	thermal conductivity	ϕ	volume fraction of the nanoparticles
L_e	Lewis number	ϕ_0^*	reference scale for the nanoparticle fraction
N_A	modified diffusivity ratio	∇_P^2	horizontal Laplacian operator
N_B	modified specific heat increment	∇^2	Laplacian operator
p	pressure		
P_r	Prandtl number	*Superscripts*	
$\vec{\mathbf{v}}_D$	Darcy velocity of nanofluid	$*$	dimensional variables
R_D	Darcy–Rayleigh number	$'$	perturbed quantities
$R_{D,c}$	critical Darcy–Rayleigh number		
t	time	*Subscripts*	
T	temperature	p	particle
(x, y, z)	space coordinates	b	basic state
		0	lower boundary
Greek symbols		1	upper boundary
β	coefficient of thermal expansion		

nanoparticles absolute velocity can be viewed as the sum of the base fluid velocity and a relative (slip) velocity. On the basis of this principle, Tzou [12], Nield and Kuznetsov [13–15], Yadav et al. [16–27], Chand and Rana [28–30] studied the problem of thermal instability in nanofluid. The common finding of these studies was that the critical Rayleigh number can be reduced or increased by a substantial amount, depending on whether the basic nanoparticle distribution is top-heavy or bottom-heavy, by the presence of nanoparticles. In studying these convective instability problems, the volume fraction of nanoparticles was prescribed at the boundaries. Two phase simulation of nanofluid flow and heat transfer was studied by Sheikholeslami et al. [31–33]. Recently, Nield and Kuznetsov [34], Yadav et al. [35–39], Shivakumara and Dhananjaya [40] and Chand and Rana [41,42] pointed out that this type of boundary condition on volume fraction of nanoparticles is physically not realistic as it is difficult to control the nanoparticle volume fraction on the boundaries and suggested the normal flux of volume fraction of nanoparticles is zero on the boundaries as an alternative boundary condition which is physically more realistic.

In actual circumstances in nanofluids, the base fluid does not satisfy the properties of Newtonian fluids; hence, it is more justified to consider them as viscoelastic fluids; for example, ethylene glycol–Al_2O_3, ethylene glycol–CuO, and ethylene glycol–ZnO are some examples of viscoelastic nanofluids. With the growing importance of viscoelastic nanofluids in technology and industries, the investigations of such fluids are desirable. Nonetheless, the study of thermal instability in a viscoelastic nanofluid saturated porous layer is relatively of recent origin and it is still in a rudimentary stage. Sheu [43], Yadav et al. [44], Umavathi et al. [45] and Shivakumara et al. [46] studied the onset of convection in a horizontal layer of porous medium saturated by a viscoelastic nanofluid using Oldroyd-B model while Rana and Chand [47] extended this study with Rivlin–Ericksen elastico-viscous model. Thermal instability of a nonhomogeneous power-law nanofluid in a porous layer with horizontal throughflow was studied by Kang et al. [48]. They obtained that the critical Rayleigh number can be significantly reduced or increased with the increasing power-law index, mainly depending on the value of Péclet number.

In the present paper, the base fluid is taken as (Kuvshiniski-type) viscoelastic fluid. To the best of our knowledge, no studies have far been investigated to analyze the onset of thermal convection of (Kuvshiniski-type) viscoelastic nanofluid layer in porous medium. The objective of the present paper was therefore to extend the previous work by taking base fluid as (Kuvshiniski-type) viscoelastic fluid considering the flux of volume fraction of nanoparticles is zero at the boundaries. The effects of the embedded flow controlling parameters on the thermal convection have been demonstrated graphically and discussed.

2. Mathematical formulation of the problem

We consider an infinite horizontal layer of incompressible nanofluid heated from below as shown in Fig. 1. A Cartesian coordinate system (x, y, z) is chosen in which z axis is taken at right angle to the boundaries. The nanofluid is confined between two parallel plates $z^* = 0$ and $z^* = L$, where the temperatures at the lower and upper boundaries are taken as T_0^* and T_1^*, respectively, T_0^* being greater than T_1^*. Asterisks are used to distinguish the dimensional variables from the non-dimensional variables (without asterisks).