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Electro-thermal convection in a Brinkman porous medium saturated by nanofluid

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KEYWORDS

Nanofluid;
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Prandtl number

Abstract Thermal instability in a horizontal layer of nanofluid with vertical AC electric field in a porous medium is investigated. Brinkman–Darcy model is used for the momentum equation. The model used for nanofluid incorporates the effect of Brownian diffusion, thermophoresis and electrophoresis. The flux of volume fraction of nanoparticles is taken to be zero on the isothermal boundaries and the eigenvalue problem is solved by using the Galerkin method. Linear stability theory based upon the normal mode technique is employed for stability analysis. Oscillatory convection has been ruled out for the problem and graphs have been plotted to study the effects of Brinkman–Darcy number, Lewis number, modified diffusivity ratio, concentration Rayleigh number, AC electric Rayleigh number and porosity parameters on stationary convection.

1. Introduction

When a small amount of nano-sized particles is added to the base fluid, the thermal conductivity of the fluid enhances and such a fluid is called nanofluid which was first coined by Choi [1]. Nanofluids have unique chemical and physical properties that make them potentially useful in many applications of heat transfer and thus considered to be the next-generation heat transfer fluids. The characteristic feature of nanofluid is thermal conductivity enhancement, a phenomenon observed by Masuda et al. [2]. Recent developments in the study of heat transfer using nanofluids should be reported by Wong and Leon [3], Yu and Xie [4], and Taylor et al. [5].

Thermal convection in nanofluids in a porous medium is an important phenomenon due to its wide ranges of applications in geophysics, food processing, oil reservoir modeling, petroleum industry, bio-mechanics, building of thermal insulations and nuclear reactors. The detailed study of thermal convection in a layer of nanofluid in porous medium based upon Buongiorno's [6] model has been given by Nield and Kuznetsov [7–9], Kuznetsov and Nield [10–12], Chand et al. [13,14], Chand and Rana [15–19], Chand [20,21], Yadav [22], Yadav et al. [23–30], Yadav and Lee [31], Yadav and Kim [32] and Rana et al. [33,34]. Recently Nield and Kuznetsov [35], Chand and Rana [36,37], Rand and Chand [38,39], Chand et al. [40], Yadav and Kim [41,42], and Yadav el al. [43] pointed out that this type of boundary condition on volume fraction of nanoparticles is physically not realistic as it is difficult to control the nanoparticle volume fraction on the boundaries and suggested the normal flux of volume fraction of nanoparticles

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Nomenclature

a	dimensionless resultant wave number
a_c	critical value of wave number
d	thickness of fluid layer
D	differential operator
Da	Darcy number
$\widetilde{Da}$	Brinkman–Darcy number
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
$\mathbf{E}$	root mean square value of the electric field
E_0	root mean square value of the electric field at $z = 0$
$\mathbf{f}_e$	force of electrical origin
$\mathbf{g}$	acceleration due to gravity
k_m	thermal conductivity
Le	Lewis number
n	growth rate of disturbances
N_A	modified diffusivity ratio
N_B	modified particle-density increment
p	pressure
Pr	Prandtl number
$\mathbf{q}$	Darcy velocity vector
Ra	thermal Rayleigh number
Ra_c	critical Rayleigh number
$(Ra)_s$	thermal Rayleigh number for stationary convection
Rae	AC electric Rayleigh number
Rn	concentration Rayleigh number
t	time
T	temperature
T_0	temperature at $z = 0$
T_1	temperature at $z = d$
Va	Vadasz number
(x, y, z)	space coordinates
u	Darcy velocity component in x-direction
v	Darcy velocity component in y-direction
w	Darcy velocity component in z-direction

Greek symbols

α	coefficient of the thermal expansion
γ	thermal expansion coefficient of dielectric constant
μ	viscosity
$\tilde{\mu}$	effective viscosity
ε	porosity
Ω	angular velocity
ρ	density of the nanofluid
ρ_0	density of nanofluid at $z = 0$
ρ_e	free charge density
ρ_p	density of nanoparticles
ρ_f	density of base fluid
$(\rho c)_p$	heat capacity of nanoparticles
φ	volume fraction of the nanoparticles
φ_0	reference volume fraction of the nanoparticles at $z = 0$
κ	thermal diffusivity
ω	dimensionless frequency
σ	thermal capacity ratio
ψ	root-mean-square value of the electric potential
$\in$	dielectric constant
$\in_0$	reference dielectric constant
∇_H^2	horizontal Laplacian operator
∇^2	Laplacian operator

Superscripts

$'$	non-dimensional variables
$''$	perturbed quantities

Subscripts

p	particle
s	steady state
f	fluid
0	lower boundary
1	upper boundary
H	horizontal plane

is zero on the boundaries as an alternative boundary condition which is physically more realistic. Under the circumstances, it is desirable to investigate convective instability problems by utilizing these boundary conditions to get meaningful insight into the problems.

Natural convection under AC/DC electric field of electrically enhanced heat transfer in fluids and possible practical applications has been reviewed by Jones [44] and Chen et al. [45]. Electrically induced convection in dielectric liquids has been the subject of investigation for many decades right from the experimental work of Gross and Porter [46]. Maekawa et al. [47] considered the convective instability problem in AC/DC electric fields. Stiles et al. [48] studied the problem of convective heat transfer through polarized dielectric liquids. They observed that the convection pattern established by the electric field is quite similar to the familiar Bénard cells in normal convection. Turnbull [49] examined the effect of dielectrophoretic forces on the Bénard instability. Recently Shivakumara et al. [50] studied the effect of velocity and temperature boundary conditions on electro-thermal convection in a rotating dielectric fluid and found that AC electric field is to enhance the heat transfer and to hasten the onset of convection. Several studies have been carried out to assess the effect of AC and DC electric fields on natural convection due to the fact that many problems of practical importance involve dielectric fluids. Many geophysical problems particularly, in the study of Earth's core where the Earth's mantle consists of conducting fluid can be simulated by this convection in the dielectric liquid. The application of the temperature gradient and the electric field may be used for heat transfer enhancement in dielectric liquids and may yield large reductions in weight and volume of heat transfer systems. This technique is attractive for aerospace cooling systems as reported by Paschkewitz and Pratt [51].

The nanodielectric fluid may be used in an electrical apparatus and other electrical equipments such as distribution transformers, regulating transformers, shunt reactors, converter transformers, instrument transformers, generating