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Mathematical propositions associated with the connectivity of architectural designs

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Abstract If there exist many computer-generated architectural designs for a given set of data, then for an architect it is difficult to single one solution out among many possibilities. In this paper, we propose a technique to refine the number of possible designs on the basis of their connectivity, which is given in terms of adjacency relations among the rooms. In addition, we present few mathematical results to study the topological properties of the architectural designs, that would also be useful for the validation of proposed technique and for the classification of different architectural designs.

1. Introduction

We know that architecture is a complex amalgamation of science and art. There are functional requirements, cultural expectations and general guidelines to follow, but within these guidelines there are still limitless possibilities. Hence, designing a house for someone can be very demanding. At the same time, it is not feasible to design a house that will suit everyone. This makes the design of houses an interesting and challenging problem which can be approached with a computer algorithm. The best that can be hoped for from an automated system is to give a variety of houses which meet general requirements, and hope that one or more of them may serve as inspiration for a client's dream house [1].

One of the initial stages of the architectural design process is concerned with the generation of planar floor plans, while satisfying the given topological and dimensional constraints. A floor plan needs to be functional in a way that it must shape the flow of traffic through the house and it has measurable traits of quality, such as efficiency in being able to get important rooms quickly or being able to light key rooms with natural light from large windows. Therefore, the *topological constraints* are generally given in terms of adjacencies between the rooms. The size of the rooms factors into the overall shape, and functionality of the house. Hence, the *dimensional constraints* involve shapes or sizes of each room. For more details regarding definitions related to floor plans, refer to [2].

Geometrically, a *floor plan* is a polygon divided by straight lines into component polygons called *rooms*. The edges forming the perimeter of each room are called *walls*. Two rooms of a floor plan are said to be *adjacent* if they share a common wall or a section of wall.

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A *graph* $G = (V, E)$ is a mathematical structure consisting of two finite sets V and E. The elements of V are called *vertices* and the elements of E are called *edges*. A *simple graph* has neither self-loops nor multi-edges. Two vertices u and v are called *adjacent* if there exists an edge between them. The readers who are not familiar with the graph theory related definitions are referred to Gross and Yellen [3], Chapter 1.

Levin [4] was first to apply graph theory to architectural designs. Cousin [5] and Friedman [6], followed Levin's lead, by looking further at graph-theoretic ideas. In the same direction, many researchers have used graph theoretical approach for the design of floor plans and proposed relevant mathematical results (refer [7–10]).

Let $R(x, y)$ denotes a rectangle with width x (dimension measured horizontally) and length y (dimension measured vertically). In 1903, Dehn [11] proved his famous result which states that *$R(x, y)$ can be tiled by (finitely many) squares if and only if y/x is a rational number*. In the literature, there exist many other mathematical results related to the architectural designs, in particular to the topological constraints set by the architects. One of the main purposes of these results is to establish the feasibility of the topological constraints (for detailed discussion see Rinsma [2]). In this paper, we also present some mathematical results which are graph theoretical and correspond to the topological properties of the rectilinear floor plans. Further, we will explain the usefulness of these results for the better understanding of the proposed solutions and for reducing the number of obtained solutions.

1.1. Rectilinear floor plans

In [12], an algorithm is presented for the construction of rectilinear floor plans while satisfying the given topological and dimensional constraints. Here, the topological constraints are provided in terms of weighted adjacency matrix[1] that consists of numbers from 0 to 10 corresponding to each pair of rooms (e.g., see Table 1). These numbers represent the probability of two rooms being adjacent, i.e., number 10 corresponds to the maximum probability for the rooms to be adjacent whereas number 0 stands for the lowest probability. The width (dimension of the room measured horizontally) and length (dimension of the room measured vertically) of the rooms represent the dimensional constraints.

The algorithm is demonstrated for the rectilinear plus-shape floor plans (see Fig. 1). Therefore, in this paper, we consider the plus-shape floor plans to illustrate few exciting topological properties of the rectilinear floor plans. Architecturally, the plus shape floor plan is very interesting and different from other floor plans because of its following geometrical and topological properties as follows:

i. it can be partitioned into 5 zones where 4 of the zones can be completely independent from each other but are well connected to the central zone (e.g., in Fig. 1, consider upper and lower zones where R_{14} can be a library and R_9 can be a playing zone; both can function simultaneously in a same building without disturbing each other),
ii. it has an option to have most of its rooms directly adjacent to the exterior while keeping some of the rooms private (e.g., most of the rooms belonging to the central zone can be used for privacy),
iii. its symmetrical nature makes it visually pleasing.

This work is concerned with the automated design of large buildings with complex and specialized programs such as hospitals where the number of rooms is generally large. To automate the process, we partition the given rectilinear shape into maximum number of rectangular zones so that each zone would not be overcrowded. Therefore, the rectilinear plus-shape polygon, as shown in Fig. 2A, is partitioned into 5 rectangles (see Fig. 2B) instead of 3 rectangles (see Fig. 2C). This implies that the plus-shape floor plan in Fig. 1 can be constructed by adjoining 5 rectangular floor plans.

A *rectangular floor plan* denoted by F^R is a floor plan in which the plan boundary and each room are rectangles. It can be generated in the following two ways:

i. *Addition:* It concerns the addition of rectangular pieces, such as tiles, to produce a rectangular plan (see Krishnamurti and Roe [14]).
ii. *Dissection:* It concerns the division of a large rectangle into smaller rectangular pieces. This process is called *rectangular dissection* (see Mitchell et al. [15], Earl [16], Flemming [17–19], Bloch and Krishnamurti [20], Bloch [21]).

The *adjacency graph* of a floor plan is a simple undirected graph, obtained by representing each of its room as a *vertex* and then drawing an *edge* between any two vertices if the corresponding rooms are *adjacent*.

The connectivity of two different floor plans made up of same rooms is measured by comparing the connectivity of their adjacency graphs, which can be computed in terms of the number of edges, diameter, average distance, number of cycles, etc. (for further details about these measures, see [22], Chapter 2). In this work, the *number of edges* of the adjacency graph is regarded as a measure of connectivity because it directly corresponds to the topological constraints that need to be satisfied for designing of a floor plan.

If two adjacency graphs have same number of vertices then the one having more edges is *more connected*. For a better understanding, consider the floor plans and their adjacency graphs in Fig. 3. We can see that both the floor plans are made up of same rooms but the numbers of edges in their adjacency graphs are 15 and 11 respectively. Hence, the floor plan in Fig. 3A is more connected than the one in Fig. 3B. For the detailed discussion about adjacency and connectivity, we refer to [12,23].

In most of the work, we found that the topological constraints are given in terms of the adjacency requirement graph, which is generally a spanning subgraph of the adjacency graph

[1] Kalay [13] (Chapter 13, Fig. 13.7) mentioned the concept of weighted matrix for the problem of *space allocation*. In this matrix, the weights give the relative importance of the proximity between the rooms which is computed on the basis of the number of trips among the rooms that gives a relation between the activities they house. For example, on the basis of number of trips, in a hospital, we prefer that the room of a nurse should be close to the room of a patient in comparison with the room of a surgeon. We consider the weighted matrix as a weighted adjacency matrix and our aim is to maximize the connectivity of the arrangement. The algorithm for forming groups on the basis of this matrix is given in [12].

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