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Radiative nanofluid flow and heat transfer over a non-linear permeable sheet with slip conditions and variable magnetic field: Dual solutions

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Abstract *Objective:* This paper addresses numerical investigation of steady, magneto-hydrodynamic boundary-layer slip flow of a nanofluid past a permeable stretching/shrinking sheet with thermal radiation using RKF45 with shooting technique. The effect of viscous dissipation, suction/injection, Brownian motion, thermophoresis, partial velocity slip and thermal slip is taken into account and controlled by the non-dimensional parameters.

Results and conclusions: The dual solutions are obtained for the skin friction, Nusselt number, temperature and nanoparticle volume fraction with pertinent parameters in the domain (χ_c, ∞) and (s_c, ∞). The study shows that the Nusselt number decreases with an increase in thermophoresis parameter Nt and thermal slip parameter δ but increases with thermal radiation R and Prandtl number Pr.

Practice implications: The present problem has numerous applications in engineering and petroleum industries such as glass blowing, annealing and thinning of copper wires. The study of radiation heat transfer plays an important role in the industrial applications at high temperature.

1. Introduction

The vast study has been carried out by several researchers in the field of boundary layer flow and convective heat transfer over a stretching/shrinking sheet due to various applications in the industries and engineering process such as glass blowing, annealing and thinning of copper wires. It is obvious that the desired quality of final sheet strongly depends on the stretching rate and the rate of cooling (heat transfer) in the process of stretching. First analysis on the boundary layer flow over a stretching sheet was studied by Crane [1]. This study is extended by many researchers to examine the various aspects of flow and heat transfer characteristics. Khan and Pop [2] studied the behavior of Nusselt number and Sherwood number for the boundary layer flow of a nanofluid over a linearly stretching sheet under the consideration of two-component model. Instead of linear stretching of sheet, the quality of sheet can also be controlled with nonlinear and exponentially

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Nomenclature

a	constant
B_0	magnetic field strength (A/m)
C	nanoparticle volume fraction
C_∞	ambient volume fraction
$B(x)$	variable magnetic field (A m^{m-2})
D_B	Brownian diffusion coefficient (m^2/s)
D_T	thermophoretic diffusion coefficient (m^2/s)
Ec	Eckert number
f	dimensionless stream function
k	thermal conductivity (W/m K)
Sc	Schmidt number
L	velocity slip factor (m)
m	power index
M	dimensionless magnetic field
N	thermal slip factor (m)
Nb	Brownian motion parameter
Nt	thermophoresis parameter
Pr	Prandtl number
R	dimensionless thermal radiation
s	mass transfer parameter
T	nanofluid temperature (K)
T_w	nanofluid temperature at sheet (K)
T_∞	ambient temperature (K)
u, v	velocity components along x- and y-axis (m/s)
u_w	velocity of sheet (m/s)
v_w	mass transfer velocity (m/s)
x, y	Cartesian coordinates (m)
q_r	radiative heat flux (W/m^2)

Greek symbol

η	similarity variable
μ	dynamic viscosity (Ns/m^2)
v	kinematic viscosity (m^2/s)
ϕ	rescaled nanoparticle volume fraction
θ	dimensionless temperature
χ	stretching/shrinking parameter
σ	electric conductivity of base fluid (S/m)
$(\rho c)_f$	heat capacity of base fluid (J/K)
$(\rho c)_p$	effective heat capacity of nanoparticle material (J/K)
σ^*	Stefan–Boltzmann constant (W m^{-2} K^{-4})
β	power-law parameter
τ_w	shear stress at surface (N/m^2)
δ	thermal slip parameter

Subscript

∞	ambient condition
w	condition on surface
r	radiation
f	base fluid
s	slip condition

stretching along with consideration of heat and mass transfer characteristics. Motivated by this concept, Cortell [3] has discussed viscous flow and heat transfer over a nonlinearly stretching sheet. Rana and Bhargava [4] have extended the idea to nanofluids and employed finite element method for the numerical computation of flow and heat transfer characteristic over a nonlinearly stretching sheet. Moreover, analytical solution of the boundary layer flow over an exponential stretching sheet has been investigated (Nadeem and Lee [5]) using homotopy analysis method.

Since last few years many researchers are attracted towards nanofluid due to its enhanced thermal conductivity as compared to base fluids that are responsible for heat transfer. Nanofluid, which was first introduced by Choi [6], is dilute suspension of nanometer sized solid particle (Cu, Al, Ag, etc.) in base fluid such as water, oil and ethylene glycol. The novel characteristics of nanofluids can be utilized to develop stable suspensions with improved heat transfer. Many researchers have tried to develop the convective transport models for nanofluid. In 2006, Buongiorno [7] has presented non-homogeneous model to understand the convective transport phenomena in nanofluid and studied seven-slip mechanisms. Among these mechanisms only Brownian diffusion and thermophoresis diffusion are found most important. These two slip mechanisms are also incorporated in the study of natural convective boundary layer flow of a nanofluid over a vertical plate by Kuznetsov and Nield [8].

The study of magnetohydrodynamic has numerous applications in engineering, agriculture and petroleum industries. The problem of natural convection under the effect of a magnetic field has also applications in geophysics and astrophysics [9]. Due to this many studies were performed with the effect of magnetic field. Fang and Zhang [10] have given exact solution for MHD flow equation of fluid over a shrinking sheet. They have reported two solution branches for $M \in (0, 1)$ but for $M = 1$ single solution branch is obtained only in case of suction and when $M > 1$ there is also single branch of solution for both suction and injection. In 2011, Hamad [11] investigated the analytical solution of electrical conducting nanofluid flow over a linearly stretching sheet under the influence of magnetic field. He found that momentum boundary layer thickness decreases but thermal boundary thickness increases with magnetic field. Rana et al. [12] presented unsteady MHD transport phenomena over a stretching sheet in a rotating nanofluid. Numerical investigation of the MHD flow and heat transfer of nanofluid between two horizontal plates in rotating system using Cu, Ag, Al_2O_3 and TiO_2 nanoparticles in water has been computed by Sheikholeslami et al. [13] and it is noticed that heat transfer is the highest for TiO_2 nanoparticles. Currently, much attention has been devoted to work in the presence of magnetic field [14–18].

Several engineering processes occur due to high temperature; therefore, the study of radiation heat transfer plays an important role in the field of equipment designing [19]. Cortell [20] analyzed the boundary layer flow and heat transfer of fluid under the consideration of thermal radiation and viscous dissipation over a nonlinear stretched sheet. This work is extended by Hady et al. [21] in nanofluid and investigated the effect of thermal radiation, viscous dissipation and nanoparticle volume fraction on velocity, temperature and the rate of heat transfer at the surface. They noticed that an increase in thermal radiation decreases temperature of nanofluid which leads

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