ARTICLE IN PRESS

Ain Shams Engineering Journal (2015) **xxx**, xxx–xxx

Ain Shams University

Ain Shams Engineering Journal

www.elsevier.com/locate/asej
www.sciencedirect.com

ENGINEERING PHYSICS AND MATHEMATICS

Fibonacci-regularization method for solving Cauchy integral equations of the first kind

Mohammad Ali Fariborzi Araghi, Samad Noeiaghdam *

Department of Mathematics, Central Tehran Branch, Islamic Azad University, Tehran, Iran

Received 10 December 2014; revised 21 July 2015; accepted 7 August 2015

KEYWORDS

Collocation method;
Fibonacci polynomials;
Regularization method;
First kind Cauchy integral equation

Abstract In this paper, a novel scheme is proposed to solve the first kind Cauchy integral equation over a finite interval. For this purpose, the regularization method is considered. Then, the collocation method with Fibonacci base function is applied to solve the obtained second kind singular integral equation. Also, the error estimate of the proposed scheme is discussed. Finally, some sample Cauchy integral equations stem from the theory of airfoils in fluid mechanics are presented and solved to illustrate the importance and applicability of the given algorithm. The tables in the examples show the efficiency of the method.

1. Introduction

The theory of Cauchy integral equations has important applications in the mathematical modeling of many scientific fields such as solid mechanics, electrodynamics and elasticity [14].

Many different methods have been developed to evaluate the approximate solution of this integral equation. Kim in [7] solved Cauchy singular integral equations by using Gaussian quadrature and considered the zeros of Chebyshev polynomials of the first and second kinds as the collocation points. Srivastav and Zhang in [17] used general quadrature-collocation nodes to solve Cauchy singular integral equation. Eshkuvatov et al. [4] discussed about the efficient approximate method to solve the characteristic equation of first kind Cauchy integral equation using the zeros of Chebyshev polynomials of the first, second, third, and fourth kinds with their corresponding weight functions.

In recent year, Fibonacci-collocation method has been applied in different works. Mirzaee et al. in [12,13] used this method for solving systems of linear Fredholm integro-differential equations and Fredholm–Volterra integral equations in two-dimensional spaces. Also, Koc et al. in [8] for solving boundary value problems and Kurt et al. in [9] solved high order linear Fredholm integro-differential-difference equations by using this method.

The regularization method was established independently by Tikhonov [18,19] and Phillips [15]. This method consists of transforming first kind integral equations to second kind equations. Also, Wazwaz in [21] and Delves in [2] used this method for solving Fredholm and Volterra integral equations of the first kind. In [1], Bougoffa et al. applied the

* Corresponding author. Tel.: +98 9143527552.
E-mail addresses: fariborzi.araghi@gmail.com, m_fariborzi@iauctb.ac.ir (M.A.F. Araghi), s.noeiaghdam.sci@iauctb.ac.ir (S. Noeiaghdam).

Peer review under responsibility of Ain Shams University.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.asej.2015.08.018
2090-4479 © 2015 Faculty of Engineering, Ain Shams University. Production and hosting by Elsevier B.V.

regularization method with parameter ϵ to solve a given Cauchy integral equations of the first kind.

In this work, by combining the regularization method with the parameter $\epsilon = 1$ and Fibonacci-collocation method, a new applicable method is presented to solve a Cauchy singular integral equation of the first kind which is nominated by Fibonacci-regularization method.

This paper is organized as follows: At first, in Section 2, some preliminaries are reminded such as the Fibonacci polynomials and Cauchy integral equations, and then in Section 3, by combining Fibonacci polynomials with regularization method a new efficient method is constructed for solving the Cauchy integral equations of the first kind. In Section 4, the error analysis of the proposed method is discussed and in the sequel in Section 5 by solving some examples, we show the accuracy of the proposed scheme. The conclusions are determined in Section 6.

2. Preliminaries

2.1. Cauchy integral equations

Let the following singular integral equation

$$\int_{-1}^{1} \frac{K_0(x,t)\varphi(t)}{t-x}dt + \int_{-1}^{1} K(x,t)\varphi(t)dt = f(x), \quad -1 < x < 1, \tag{1}$$

where $K_0(x,t)$, $K(x,t)$ and $f(x)$ are given real valued functions belonging to the Holder class and $\varphi(t)$ is unknown. If in Eq. (1), $K_0(x,t) = 1$ and $K(x,t) = 0$ then

$$\int_{-1}^{1} \frac{\varphi(t)}{t-x}dt = f(x), \quad -1 < x < 1, \tag{2}$$

which is called the characteristic singular integral equation. It is well known that the analytical solutions of Eq. (2), in the following four cases, can be determined [4,10]:

Case (I): The solution is unbounded at both end-points $x = \pm 1$,

$$\varphi(x) = -\frac{1}{\pi^2\sqrt{1-x^2}} \int_{-1}^{1} \frac{\sqrt{1-t^2}f(t)}{t-x}dt + \frac{c}{\pi\sqrt{1-x^2}},$$

where

$$\int_{-1}^{1} \varphi(t)dt = c.$$

Case (II): The solution is bounded at both end-points $x = \pm 1$,

$$\varphi(x) = -\frac{\sqrt{1-x^2}}{\pi^2} \int_{-1}^{1} \frac{f(t)}{\sqrt{1-t^2}(t-x)}dt,$$

provided that

$$\int_{-1}^{1} \frac{f(x)}{\sqrt{1-t^2}}dt = 0.$$

Case (III): The solution is bounded at the point $x = -1$,

$$\varphi(x) = -\frac{1}{\pi^2}\sqrt{\frac{1+x}{1-x}} \int_{-1}^{1} \sqrt{\frac{1-t}{1+t}}\frac{f(t)}{t-x}dt.$$

Case (IV): The solution is bounded at the point $x = 1$,

$$\varphi(x) = -\frac{1}{\pi^2}\sqrt{\frac{1-x}{1+x}} \int_{-1}^{1} \sqrt{\frac{1+t}{1-t}}\frac{f(t)}{t-x}dt.$$

2.2. The Fibonacci polynomials and their properties

Leonardo of Pisa also known as Leonardus Pisanus, or, most commonly, Fibonacci (from filius Bonacci), was an Italian mathematician of the 13th century. Fibonacci is the best known to the modern world for the spreading of the Hindu–Arabic numerical system in Europe, primarily through the publication in 1202 of his Liber Abaci (Book of Calculation), and for a number sequence named the Fibonacci numbers after him, which he did not discover but used as an example in the Liber Abaci. In Fibonacci's Liber Abaci book, chapter 12, he posed, and solved, a problem involving the growth of a population of rabbits based on idealized assumptions. The solution, generation by generation, was a sequence of numbers later known as Fibonacci numbers. The number sequence was known to Indian mathematicians as early as the 6th century, but it was Fibonaccis Liber Abaci that introduced it to the West. In the Fibonacci sequence of numbers, each number is the sum of the previous two numbers, starting with 0 and 1. This sequence begins $0, 1, 2, 3, 5, \ldots$ [6,13].

Definition 1. For any positive real number k, the k-Fibonacci sequence, say $\{F_{k,n}\}_{n\in N}$ is defined recurrently by

$$F_{k,n+1} = kF_{k,n} + F_{k,n-1}, \quad n \geqslant 1,$$

with initial conditions

$$F_{k,0} = 0, \quad F_{k,1} = 1.$$

Particular cases of the k-Fibonacci sequence are constructed from the following relations

if $k = 1$, the classical Fibonacci sequence is obtained:

$$F_0 = 0, \quad F_1 = 1, \quad F_{n+1} = F_n + F_{n-1}, \quad n \geqslant 1,$$

if $k = 2$, the Pell sequence appears:

$$P_0 = 0, \quad P_1 = 1, \quad P_{n+1} = 2P_n + P_{n-1}, \quad n \geqslant 1,$$

if $k = 3$, the following sequence appears:

$$H_0 = 0, \quad H_1 = 1, \quad H_{n+1} = 3H_n + H_{n-1}, \quad n \geqslant 1.$$

If k be a real variable x then $F_{k,n} = F_{x,n}$ and they correspond to the Fibonacci polynomials defined by

$$F_{n+1}(x) = \begin{cases} 1, & n = 0, \\ x, & n = 1, \\ xF_n(x) + F_{n-1}(x) & n > 1, \end{cases}$$

from where the first five Fibonacci polynomials are

$$F_1(x) = 1,$$
$$F_2(x) = x,$$
$$F_3(x) = x^2 + 1,$$
$$F_4(x) = x^3 + 2x,$$
$$F_5(x) = x^4 + 3x^2 + 1,$$