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Mixed convection boundary layer flows of a non-Newtonian Jeffrey's fluid from a non-isothermal wedge

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Abstract This article presents the nonlinear, steady state mixed convection boundary layer flow, heat and mass transfer of an incompressible non-Newtonian Jeffrey's fluid past a non-isothermal wedge. The transformed conservation equations are solved numerically subject to physically appropriate boundary conditions using a versatile, implicit finite-difference Keller box technique. The influence of a number of emerging non-dimensional parameters, namely Deborah number (De), ratio of relaxation to retardation times (λ), pressure gradient parameter (m), Buoyancy ratio parameter (N), mixed convection parameter (λ_1), radiation parameter (F) and heat generation/absorption parameter (Δ) on velocity, temperature and concentration evolution in the boundary layer regime is examined in detail. Also, the effects of these parameters on *surface heat transfer rate*, *mass transfer rate* and *local skin friction* are investigated.

1. Introduction

Non-Newtonian transport phenomena arise in many branches of process mechanical, chemical and materials engineering. Most non-Newtonian models involve some form of modification to the momentum conservation equations. These include power-law fluids [1], Walters-B short memory models [2,3], Oldroyd-B models [4], differential Reiner–Rivlin models [5,6], Bingham plastics [7] and tangent hyperbolic fluids [8].

As with a number of rheological models developed, the Jeffrey model has proved quite successful. This simple, yet elegant rheological model was introduced originally to simulate earth crustal flow problems [9]. This model [10] constitutes a viscoelastic fluid model which exhibits shear thinning characteristics, yield stress and high shear viscosity. The Jeffrey fluid model degenerates to a Newtonian fluid at a very high wall shear stress i.e. when the *wall stress is much greater than yield stress*. This fluid model also approximates reasonably well the rheological behavior of other liquids including physiological suspensions, foams, geological materials, cosmetics, and

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Nomenclature

C	concentration
C_f	skin friction coefficient
c_p	specific heat parameter
De	Deborah number
D_m	mass (species) diffusivity
f	non-dimensional steam function
F	thermal Radiation
$\mathbf{g}$	acceleration due to gravity
Gr_x	Grashof (free convection) number
K	thermal diffusivity
k	thermal conductivity of Jeffrey's fluid
k^*	mean absorption coefficient
m	pressure gradient parameter
Nb	buoyancy ratio parameter
Nu	heat transfer rate (local Nusselt number)
Pr	Prandtl number
q_r	radiative heat flux
Re_x	Reynolds number
$\mathbf{S}$	Cauchy stress tensor
Sc	local Schmidt number
Sh	mass transfer rate (Sherwood number)
T	temperature of the Jeffrey's fluid
u, v	non-dimensional velocity components along the x- and y-directions, respectively
x	stream wise coordinate
y	transverse coordinate

Greek symbols

α	thermal diffusivity
β	coefficient of thermal expansion
β^*	coefficient of concentration expansion
λ	ratio of relaxation to retardation times
λ_1	mixed convection parameter
λ_2	retardation time
η	dimensionless radial coordinate
μ	dynamic viscosity
v	kinematic viscosity
θ	non-dimensional temperature
ϕ	non-dimensional concentration
ρ	density of fluid
ξ	dimensionless tangential coordinate
Ψ	dimensionless stream function
Δ	heat generation (source)/heat absorption (sink) parameter
σ^*	Stefan–Boltzmann constant

Subscripts

w	surface conditions on wedge
∞	free stream conditions

syrups. Interesting studies employing this model include peristaltic transport of Jeffery fluid under the effect of magnetohydrodynamic [11], peristaltic flow of Jeffery fluid with variable-viscosity [12], radiative flow of Jeffery fluid in a porous medium with power law heat flux and heat source [13], recent studies on Jeffrey's fluid include [14–16].

The heat transfer analysis of boundary layer flow with radiation is important in various materials processing operations including high temperature plasmas, glass fabrication, and liquid metal fluids. When coupled with thermal convection flows, these transport phenomena problems are highly nonlinear. At a high temperature the presence of thermal radiation changes the distribution of temperature in the boundary layer, which in turn affects the heat transfer at the wall. A number of studies have appeared that consider multi-physical radiative–convective flows. Recently, Asmat Ara et al. [17] reported the radiation effect on boundary layer flow of Eyring-Powell fluid from an exponentially shrinking sheet. Noor et al. [18] used the Rosseland model to study radiation effects on hydromagnetic convection with thermophoresis along an inclined plate. Further, studies employing the Rosseland model include Gupta et al. [19] who examined on radiative convective micropolar shrinking sheet flow, Cortell and Suction [20] who investigated non-Newtonian dissipative radiative flow, and Bhargava et al. [21] who studied radiative-convection micropolar flow in porous media.

Very few of the above studies have considered *Falkner–Skan flows* [22,23]. This family of boundary layer flows is associated with the two-dimensional wedge configuration. Non-Newtonian flows from wedge bodies arise in a number of chemical engineering systems which have been described in detail by Peddieson [24] employing the second order Reiner–Rivlin model. The mixed convection boundary layer flow from a heated wedge plate has also drawn some interest. The combined forced and free convection flow and heat transfer about a non-isothermal wedge subject to a non-uniform free stream velocity was first considered by Sparrow et al. [25]. Watanabe et al. [26] analyzed theoretically mixed convection flow over a perforated wedge with uniform suction or injection. Kafoussias and Nanousis [27] and Nanousis [28] studied the effect of suction or injection on MHD mixed convection flow past a wedge. Gorla [29] used a power-law model to study heat transfer in polymer flow past a wedge. Yih [30] evaluated radiation effects on mixed convection flow about an isothermal wedge embedded in a saturated porous medium. Rashidi et al. [31] developed homotopy solutions for third grade viscoelastic flow from a non-isothermal wedge. Chamkha et al. [32] presented computational solutions for MHD forced convection flow from a non-isothermal wedge in the presence of a heat source or sink with a finite difference method. Hsiao [33] reported on MHD convection of viscoelastic fluid past a porous wedge, observing that the elastic effect increases the local heat transfer coefficient and heat transfer rates at the wedge surface. Ishak et al. [34] obtained a self-similar solution for a moving wedge in a micropolar fluid. Ishak et al. [35] further studied numerically steady two-dimensional laminar flow past a moving wedge in non-Newtonian fluid. Ishak et al. [36] studied the MHD boundary layer flow of a micropolar fluid past a wedge with constant heat flux. Ishak et al. [37] examined the MHD boundary layer flow of a micropolar fluid past a wedge with variable wall temperature.

The current work presents a numerical study of laminar boundary layer flow, heat and mass transfer of Jeffrey's

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