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MHD natural convection flow in a vertical micro-concentric-annuli in the presence of radial magnetic field: An exact solution

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Volume flow rate

Abstract This paper deals with a theoretical investigation of steady fully developed MHD natural convection flow of viscous, incompressible, electrically conducting fluid in micro-concntric-annuli in the presence of radial magnetic field. The velocity slip and temperature jump at the micro-concentric annuli surface are taken into account. Exact solutions are derived for energy and momentum equations under relevant boundary conditions. The solution obtained is graphically represented and the effects of various controlling parameters such as the radius ratio (η), Hartmann number (M), rarefaction parameter ($\beta_v Kn$), and fluid-wall interaction parameter (F) on the flow formation are discussed. The significant result from the study is that as rarefaction parameter ($\beta_v Kn$) increases the velocity slip on the surface of cylinders increases while fluid wall interaction parameter (F) decreases the velocity inside the micro-concntric-annuli. Furthermore, it is found that increase in radius ratio leads to increase in the volume flow rate.

1. Introduction

One of the major themes in science and technology during the past half century has been miniaturization down to the micro- and nanoscale. The area of micro and nanofluidics is fundamentally important due to the necessity of understanding the nature of fluid flow at this scale [1]. Free-convective gas micro flow, encountered in many engineering fields, i.e., micro electrochemical cell transport, micro heat exchanging, and microchip cooling, is an attractive branch of micro fluidics [2] due to its reliability, simplicity, and cost effectiveness in flow and heat transfer mechanism. Chen and Weng [3] analytically studied the fully developed natural convection in open-ended vertical parallel plate micro-channel with asymmetric wall temperature distribution in which the effect of rarefaction and fluid wall interaction was shown to increase the volume flow rate and decrease the heat transfer. This result is further extended by taking into account suction/injection on the micro-channel walls by Jha et al. [4]. They concluded that skin

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Nomenclature

H_0	constant strength of applied magnetic field
F	fluid-wall interaction parameter, β_t/β_v
g	gravitational acceleration
k_1	radius of the inner cylinder
k_2	radius of the outer cylinder
Kn	Knudsen number, λ/w
M	Hartmann number
q	volume flow rate
Q	dimensionless volume flow rate
Pr	Prandtl number
r	dimensional radial coordinate
R	dimensionless radial coordinate
$\widehat{R}$	specific gas constant
T	temperature of fluid
T_0	reference temperature
T_1	temperature at outer surface of the inner cylinder
u	axial velocity
U	dimensionless axial velocity
w	k_2–k_1
σ_t, σ_v	thermal and tangential momentum accommodation coefficients, respectively

Greek letters

β	coefficient of thermal expansion
β_t, β_v	dimensionless variables
γ	ratio of specific heats (C_ρ/C_v)
θ	dimensionless temperature
ρ	density
μ_e	magnetic permeability
v	fluid kinematic viscosity
η	ratio of radii (k_1/k_2)
λ	molecular mean free path
k	thermal conductivity
σ	electrical conductivity of the fluid

frictions as well as the rate of heat transfer are strongly dependent on suction/injection parameter. In an another work, Weng and Chen [5] studied the impact of wall surface curvature on steady fully developed natural convection flow in an open-ended vertical micro-annulus with an asymmetric heating of annulus surface. Recently, Jha et al. [6] further extended the work of Weng and Chen [5] by taking into account suction/injection on vertical annular micro-channel. In a related article, Avci and Aydin [7] studied the fully developed mixed convective heat transfer of a Newtonian fluid in a vertical micro-annulus between two concentric micro-tubes. Recently, Jha and Aina [8] further extended the work of Avci and Aydin [7] to the case when the vertical micro-annulus formed by two concentric micro-tubes is porous, i.e. where there is suction or injection of fluid through the annulus surfaces.

The annular geometry is widely employed in the field of heat exchangers. Its typical application is the gas cooled nuclear reactors in which the cylindrical fissionable fuel elements are placed axially in vertical coolant channel within the graphite moderators and the cooling gas is flowing along the channel parallel to the fuel elements [9]. The practical application of flow in annular space can also be found for example in drilling operation of oil and gas wells. The MHD flow problem in an annulus was first discussed by Globe [10] who considered fully developed laminar MHD flow in an annular channel. Jain and Mehta [11] extended the problem by imposing suction/injection on the walls. An exact solution of electrically conducting viscous incompressible flow in an annulus with porous wall under an external radial magnetic field was obtained by Nandi [12]. Antimirov and Kolyshkin [13] studied the unsteady magneto hydrodynamic flow in an annular channel with radial magnetic field while Takhar and Ali [14] examined the stability of MHD Couette flow in a narrow gap annulus. In a recent study, Jha et al. [15] investigated the influence of externally applied transverse magnetic field on steady natural convection flow of conducting fluid in a vertical micro-channel while Jha et al. [16] obtained an exact solution of steady fully developed natural convection flow of viscous, incompressible, electrically conducting fluid in a vertical annular micro-channel with the effect of transverse magnetic field. Sheikholeslami et al. [17] investigated the magnetic field effect on nanofluid flow and heat transfer in a semi-annulus enclosure via control volume based finite element method. Khan and Ellahi [18] observed the effects of magnetic field and porous medium on some unidirectional flows of a second grade fluid. Farhad et al. [19] examined the slip effect on hydromagnetic rotating flow of viscous fluid through a porous space. In another work, Farhad et al. [20] investigated the effects of slip condition on the unsteady magnetohydrodynamics (MHD) flow of incompressible viscoelastic fluids in a porous channel under the influence of transverse magnetic field and Hall current with heat and mass transfer. An analysis to investigate the combined effects of heat and mass transfer on free convection unsteady magnetohydrodynamics (MHD) flow of viscous fluid embedded in a porous medium is presented by Farhad et al. [21].

Some recent works related to the present investigation are found in the literature [22–27]. Avci and Aydin [22] analyzed analytically the forced convection heat transfer in fully developed flows of viscous dissipating fluids in concentric annular ducts. In [23], the hydrodynamically and thermal fully developed flows of viscous dissipating gases in annular micro-duct between two concentric cylinders are analyzed analytically. In another work, El-Shaarawi and Al-Nimr [24] considered the fully developed natural convection in open-ended vertical concentric annuli. Analytical solutions for transient fully developed natural convection in open-ended vertical concentric annuli are presented by Al-Nimr [25]. Also, Al-Nimr [26] carried out analytical solutions for fully developed MHD natural-convection flow in open-ended vertical concentric porous annuli. Al-Nimr and Darabseh [27] presented the closed forms on transient fully developed free convection solutions, corresponding to four fundamental thermal boundary conditions in vertical concentric annuli.

The objective of this present work is to investigate the influence of wall surface curvature on steady fully developed MHD natural convection flow of viscous, incompressible, electrically conducting fluid in micro-concentric annuli under a radial

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