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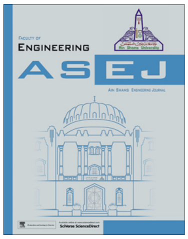

ENGINEERING PHYSICS AND MATHEMATICS

Thermo-diffusive Darcy flow induced by a concentrated source

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Abstract An analytic study is made of Soret-induced double diffusive Darcy flow produced in an unbounded homogeneous porous medium of uniform porosity and low permeability when a concentrated source embedded instantaneously in the medium starts liberating heat and at the same time a chemical substance too at a constant rate in a regime where the temperature gradient produces mass flux as well. A perturbation analysis in the limit of small Rayleigh number is employed to obtain analytical solution for the determination of the transient and steady-state development of the flow field and heat and mass transfer. Due to double diffusion, a bifurcation of the flow field is noticed when the buoyancy mechanisms are opposed and due to the Soret-induced cross-diffusion, the region in which the thermal effect of the source is felt, gets minimized with a simultaneous reduction in the rate of momentum and heat transfer.

1. Introduction

In view of the wide ranging applications to a variety of fields such as the fluid dynamics of weak thermal explosions and cooling of the components of electrical and electronic equipments, several researchers have investigated the phenomenon of the buoyancy-driven thermal convection due to the presence of point sources embedded in saturated porous media and have obtained several results for the flow and temperature fields assuming the validity of the Darcy and extended Brinkman models in both infinite and semi-infinite media. One of the earliest works being that of Wooding [1], who studied the free convection in a saturated porous medium at large Rayleigh number or Peclet number by developing a boundary-layer theory for the analysis of vertical plane flows in regions where the gradients of fluid properties are very large. Bejan [2] investigated in the low Rayleigh number regime, the natural convection flow and heat transfer around a concentrated point source embedded in an unbounded porous medium using a regular perturbation. Using similarity transformations, Hickox and Watts [3] obtained a numerical solution for the steady, axi-symmetric velocity and temperature fields associated with a point source of thermal energy while, Hickox [4] investigated an identical problem with a special emphasis on its applications

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Nomenclature

A	non-dimensional parameter, $\{(\varepsilon/\sigma)Le\}^{1/2}$
C	the species concentration
C^*	non-dimensional species concentration
c_p	specific heat at constant pressure
D_{CT}	thermo-diffusion coefficient
D_m	solutal diffusivity
E^2	$(\partial/\partial\eta)\{\eta^2 \cdot \sin\varphi(\partial/\partial\eta)\} + (\partial/\partial\varphi)\{\sin\varphi \cdot (\partial/\partial\varphi)\}$ (non-dimensional operator)
$\mathbf{e}_{1,2,3}$	unit vectors in the increasing directions of r, φ, and θ
F	function of (η, φ) (Eq. (8))
f	function of η (Eq. (24))
G	function of (η, φ) (Eq. (8))
g	acceleration due to gravity
g_1	function of η, Eq. (30)
H	function of (η, φ) (Eq. (8))
h_1	function of η, Eq. (30)
K	permeability of the porous medium
k	thermal conductivity of the fluid-porous matrix
Le	Lewis number (α/D_m)
m	species generation rate
N	buoyancy ratio $(\beta_c mk/(\beta Q D_m)$
$\boldsymbol{n}$	unit vector along the $\varphi = 0$ axis
Q	thermal energy of the source
$\boldsymbol{q}$	the mean filtration velocity of the fluid in the medium
R	non-dimensional radial co-ordinate
R_a	thermal Rayleigh number $(\beta g K/\alpha\nu k)Q$
r	radial co-ordinate
T	temperature
t	time

Greek symbols

α	effective thermal diffusivity of the fluid-porous matrix
β	thermal expansion coefficient
β_C	solutal expansion coefficient
δ	integration parameter, Eq. (30)
λ	Soret parameter (QD_{CT}/km)
ε	porosity of the porous matrix
ζ	$\cos\varphi$
$\zeta_{1,2}$	functions of η (Eqs. (32)–(33))
η	similarity variable, $R/2\sqrt{\tau}$
θ	polar angle
Θ	non-dimensional temperature
μ	coefficient of viscosity of the fluid
ν	kinematic viscosity of the fluid
ξ	function of η, Eq. (27)
ρ	fluid density
σ	heat capacity ratio, $\varepsilon + (1-\varepsilon)(\rho c_p)_s/(\rho c_p)_f$
τ	non-dimensional time
Ω	function of η, Eq. 33(c)
φ	cone angle
χ	function of η, Eq. (25)
Ψ	non-dimensional stream function
ψ	stream function
ω	non-dimensional parameter, $\lambda/(1-A^2)$
∇	vector operator, $\mathbf{e}_1\partial/\partial r + \mathbf{e}_2 r^{-1}\partial/\partial\varphi + \mathbf{e}_3(r^2\sin\varphi)^{-1}\,\partial/\partial\theta$
Δ	differential operator $r^{-2}\{(\partial/\partial r)(r^2\partial/\partial r) + (\sin\varphi)^{-1}(\partial/\partial\varphi)(\sin\varphi \cdot \partial/\partial\varphi)\}$

Subscripts

∞	reference state
k	kth term $(k = 0, 1, 2, \ldots)$
m	maximum value
n	nth term $(n = 0, 1, 2, \ldots)$
f	fluid phase
s	solid phase

to sub-seabed disposal of nuclear waste. Natural convection from a point source embedded in a Darcian porous medium bounded by an adiabatic conical surface was studied by Afzal and Salam [5]. The development of the flow field and temperature induced by a pulsating point heat source was studied by Purushothaman et al. [6], who obtained among other things, an analytical solution for the second-order mean flow. Thermal convection at low Rayleigh number induced by an instantaneous point heat source in a non-Darcy porous medium was studied by Ganapathy and Purushothaman [7]. The Brinkman term was found to affect the solution at radial distances up to $O(\sqrt{K})$ from the source. Ganapathy [8] treated an instantaneous point source which is enveloped by a solid sphere which is itself surrounded by a porous medium. Recently, Ganapathy and Mohan [9] investigated the non-linear Darcy flow and obtained an analytical solution for the flow of water at 4 °C in a low Rayleigh number regime.

With the knowledge accumulated from studies on convective heat transfer due to the presence of heat sources in porous media, considerable attention is now being paid on more sophisticated problems that lay stress on mass transfer effects. In fact, in most practical situations, quite often, species concentration gradients greatly affect the flow and as a result they play a decisive role in the development of the flow and thermal fields. In view of its wide ranging applications to energy related engineering problems such as migration of moisture in fibrous insulations, spreading of pollutants in water saturated soil and underground disposal of nuclear waste, considerable work has been done and several results are available in the literature on such thermohaline convection. However, with the possible exception of the works of Poulikakos [10] and Ganapathy [11], most of the existing studies on such thermohaline convection in spherical geometry, are primarily concerned with spherical sources and heated spheres only (for instance, Ganapathy and Mohan [12], Ganapathy [13]), Lai and Kulaki [14]) and those concerned with concentrated point sources seem to be limited in the literature. While Poulikakos [10] reports an analytical study on the buoyancy induced heat and mass transfer from a concentrated source in an infinite porous medium assuming the Darcy flow model, Ganapathy [11] dealt with an identical problem of penetrative convection in a non-Darcy medium assuming the validity of the Brinkman model.

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