ARTICLE IN PRESS

Ain Shams Engineering Journal (2015) xxx, xxx–xxx

Ain Shams University

Ain Shams Engineering Journal

www.elsevier.com/locate/asej
www.sciencedirect.com

MECHANICAL ENGINEERING

Analytical approach for solving two-dimensional laminar viscous flow between slowly expanding and contracting walls

E. Rahimi [a], A. Rahimifar [b], R. Mohammadyari [c], I. Rahimipetroudi [d], M. Rahimi-Esbo [c,*]

[a] *Department of Mechanical Engineering, Science and Research Branch, Islamic Azad University, Arak, Iran*
[b] *Department of Mechanical Engineering, Babol University of Technology, Babol, Iran*
[c] *Department of Mathematics, Buinzahra Branch, Islamic Azad University, Buinzahra, Iran*
[d] *Young Researchers Club, Sari Branch, Islamic Azad University, Sari, Iran*

Received 20 February 2015; revised 21 June 2015; accepted 23 July 2015

KEYWORDS

Homotopy Analysis Method;
Homotopy Perturbation Method;
Viscous flow;
Weak permeability

Abstract In this article, an analysis has been performed to study the two dimensional viscous flow between slowly expanding and contracting walls with weak permeability. The governing equations for the base fluid of this problem are described by dimensionless parameters wall dilation rate (α) and permeation Reynolds number (*Re*). The nonlinear differential equation is solved using two different analytically approaches by Homotopy Analysis Method (HAM) and Homotopy Perturbation Method (HPM). Then, the results are compared with numerical solution by fourth order Runge–Kutta–Fehlberg technique. Furthermore, the effects of dimensionless parameters on the velocity distributions are investigated in this research. As an important outcome, it is observed that, great agreement was found between the obtained results from the analytical and the numerical models.

1. Introduction

The flow of Newtonian and non-Newtonian fluids in a porous surface channel has attracted the interest of many investigators in view of its applications in engineering practice, particularly in chemical industries. Examples of these are the cases of boundary layer control, transpiration cooling and gaseous diffusion. Theoretical research on steady flow of this type was initiated by Berman [1] who found a series solution for the two-dimensional laminar flow of a viscous incompressible fluid in a parallel-walled channel for the case of a very low cross-flow Reynolds number. After his work, this problem has been studied by many researchers considering various variations in the problem, e.g., Choi et al. [2] and references cited therein. For the case of a converging or diverging channel with a permeable wall, if the Reynolds number is large and if

* Corresponding author. Tel.: +98 9116277073.
E-mail address: rahimi.mazaher@gmail.com (M. Rahimi-Esbo).
Peer review under responsibility of Ain Shams University.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.asej.2015.07.013
2090-4479 © 2015 Faculty of Engineering, Ain Shams University. Production and hosting by Elsevier B.V.

there is suction or injection at the walls whose magnitude is inversely proportional to the distance along the wall from the origin of the channel, a solution for laminar boundary layer equations can be obtained [3]. An interesting subject has been carried out by different authors [4–9].

Most of problems and scientific phenomena such as heat transfer are inherently of nonlinearity. We know that except a limited number of these problems, most of them do not have exact solutions. Therefore, these nonlinear equations should be solved approximately either numerically or analytically. In the numerical method, stability and convergence should be considered so as to avoid divergence or inappropriate results. Time consuming is another problem of numerical techniques. Analytical solutions often fit under classical perturbation methods. Perturbation method [10] provides the most versatile tools available in nonlinear analysis of engineering problem, but its limitations restrict its application [11,12]:

Perturbation method is based on assuming a small parameter. The majority of nonlinear problems, especially those having strong nonlinearity, have no small parameters at all. The approximate solutions obtained by the perturbation methods, in most cases, are valid only for small values of the small parameter. Generally, the perturbation solutions are uniformly valid as long as a scientific system parameter is small. However, we cannot rely fully on the approximations, because there is no criterion on which the small parameter should exist. Thus, it is essential to check the validity of the approximations numerically and/or experimentally. To overcome these difficulties, some new methods have been proposed such as VIM, HPM, and ADM. Disappointingly, the majority of nonlinear problems have no small parameter at all. Recently, several new techniques have been presented to overcome the mentioned difficulties. Some of these techniques include Variational Iteration Method (VIM) [13,14], decomposition method [15], Homotopy Perturbation Method (HPM) [16,17] and Homotopy Analysis Method [18–26].

In this letter, analytical solutions of nonlinear equations arising of two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability have been studied by three analytical methods. These methods called HAM and HPM that which does not small parameter. Obtaining the analytical solution of the models and comparing with numerical result reveal the capability, effectiveness and convenience of HAM and HPM. These methods give successive approximations of high accuracy solution.

2. Governing equations

Consider the laminar, isothermal, and incompressible flow in a rectangular domain bounded by two permeable surfaces that enable the fluid to enter or exit during successive expansions or contractions [27]. A schematic diagram of the problem is shown in Fig. 1. Both walls are assumed to have equal permeability and to expand uniformly at a time dependent rate $\dot{a}$. Furthermore, the origin $x^* = 0$ is assumed to be the center of the classic squeeze film problem. This enables us to assume flow symmetry about $x^* = 0$. Under these assumptions, the equations for continuity and motion become

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0, \tag{1}$$

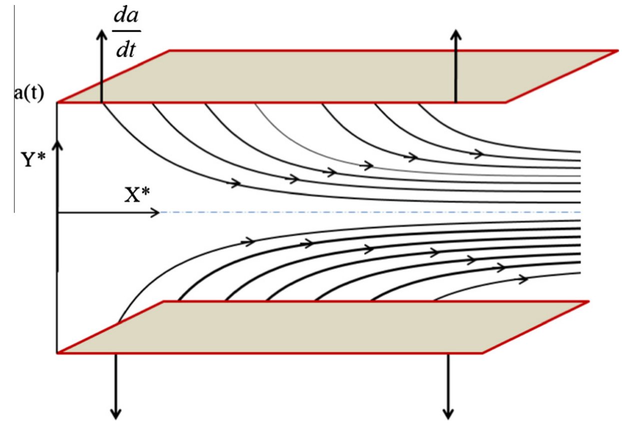

Figure 1 Two-dimensional domain with expanding or contracting porous walls.

$$\frac{\partial u^*}{\partial t} + u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial x^*} + \upsilon\nabla^2 u^*, \tag{2}$$

$$\frac{\partial v^*}{\partial t} + u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial \widehat{p}}{\partial y^*} + \upsilon\nabla^2 v^*. \tag{3}$$

In the above equations u^* and v^* indicate the velocity components in x^* and y^* directions, P^* denotes the dimensional pressure, ρ, υ and t are the density, kinematic viscosity, and time. Auxiliary conditions can be specified as follows:

$$y^* = a(t): \quad u^* = 0, \quad v^* = -v_w = -\frac{\dot{a}}{c}, \tag{4}$$

$$\begin{aligned} y^* = 0: &\quad \frac{\partial u^*}{\partial y^*} = 0, \quad v^* = 0, \\ x^* = 0: &\quad u^* = 0, \end{aligned} \tag{5}$$

where c ($c \equiv \dot{a}/v_w$) is the wall permeance or injection/suction coefficient, that is a measure of wall permeability. At this point, the stream function and mean flow vorticity can be introduced by putting [27]:

$$u^* = \frac{\partial \psi^*}{\partial y^*}, \quad v^* = \frac{\partial \psi^*}{\partial x^*}, \tag{6}$$

$$\xi^* = \frac{\partial v^*}{\partial x^*} - \frac{\partial u^*}{\partial y^*}, \tag{7}$$

$$\frac{\partial \xi^*}{\partial t} + u^*\frac{\partial \xi^*}{\partial x^*} + v^*\frac{\partial \xi^*}{\partial y^*} = \upsilon\left[\frac{\partial^2 \xi^*}{\partial x^{*2}} + \frac{\partial^2 \xi^*}{\partial y^{*2}}\right]. \tag{8}$$

Substituting Eq. (7) into Eq. (8) yields

$$\begin{aligned} &v^*_{xt} - u^*_{yt} + u^*(v^*_{xx} - u^*_{yx}) + v^*(v^*_{xy} - u^*_{yy}) \\ &\quad = \upsilon(v^*_{xxx} - u^*_{yxx} + v^*_{xyy} - u^*_{yyy})). \end{aligned} \tag{9}$$

Due to mass conservation, a similar solution can be developed with respect to x^* starting with [28]:

$$\begin{aligned} &\psi^* = \frac{\upsilon x^* f^*(y,t)}{a}, \quad u^* = \upsilon x^* a^{-2} f^*_y, \quad v^* = -\upsilon a^{-1} f^*(y,t), \\ &y = \frac{y^*}{a}, \quad f^*_y = \frac{\partial f^*}{\partial y}. \end{aligned} \tag{10}$$