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ORIGINAL ARTICLE

An algorithm based on a new DQM with modified extended cubic B-splines for numerical study of two dimensional hyperbolic telegraph equation

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Abstract In this paper, a new approach "modified extended cubic B-Spline differential quadrature (mECDQ) method" has been developed for the numerical computation of two dimensional hyperbolic telegraph equation. The mECDQ method is a DQM based on modified extended cubic B-spline functions as new base functions. The mECDQ method reduces the hyperbolic telegraph equation into an amenable system of ordinary differential equations (ODEs), in time. The resulting system of ODEs has been solved by adopting an optimal five stage fourth-order strong stability preserving Runge - Kutta (SSP-RK54) scheme. The stability of the method is also studied by computing the eigenvalues of the coefficient matrices. It is shown that the mECDQ method produces stable solution for the telegraph equation. The accuracy of the method is illustrated by computing the errors between analytical solutions and numerical solutions are measured in terms of L_2 and L_∞ and average error norms for each problem. A comparison of mECDQ solutions with the results of the other numerical methods has been carried out for various space sizes and time step sizes, which shows that the mECDQ solutions are converging very fast in comparison with the various existing schemes.

1. Introduction

The hyperbolic partial differential equation plays an important role in modeling various fundamental equations in atomic physics [1] and is very useful in understanding various physical phenomena in applied sciences and engineering. These equations are also used in the modeling of the vibrations of structures, for example, in buildings, machines and beams. The telegraph equation is more convenient than the ordinary diffusion equation in modeling reaction-diffusion for such branches of sciences [2], and mostly used in wave propagation of electric signals in a cable transmission line [3].

Consider second-order two-space dimensional linear hyperbolic telegraph equation of the form

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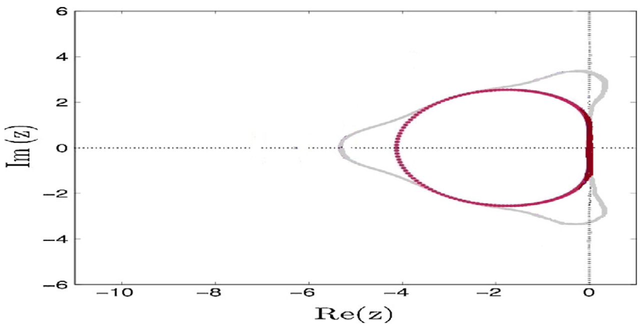

Figure 1 The stability region of SSP-RK54 scheme.

$$\frac{\partial^2 u(x,y,t)}{\partial t^2} + 2\alpha \frac{\partial u(x,y,t)}{\partial t} + \beta^2 u(x,y,t) = \frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2} + f(x,y,t), \quad (x,y) \in \Omega, t > 0, \tag{1}$$

where $\partial\Omega$ denotes the boundary of the computational domain $\Omega = [0,1] \times [0,1] \subset R^2$ and $\alpha > 0, \beta$ are arbitrary constants. Eq. (1) with $\beta = 0$ is a damped wave equation while for $\beta > 0$ it reduces to telegraph equation.

Two space dimensional initial boundary value problem for second order linear telegraph Eq. (1) is obtained by combining this equation with the following initial conditions:

$$u(x,y,0) = \phi(x,y), \qquad u_t(x,y,0) = \psi(x,y), \qquad (x,y) \in \Omega, \tag{2}$$

and the boundary conditions: (a) Dirichlet boundary conditions

$$\begin{cases} u(0,y,t) = \phi_1(y,t), u(1,y,t) = \phi_2(y,t), \\ u(x,0,t) = \phi_3(x,t), u(x,1,t) = \phi_4(x,t), \end{cases} \quad (x,y) \in \partial\Omega, t > 0 \tag{3}$$

or (b) Neumann boundary conditions

$$\begin{cases} u_x(0,y,t) = \psi_1(y,t), u(1,y,t) = \psi_2(y,t), \\ u_y(x,0,t) = \psi_3(x,t), u(x,1,t) = \psi_4(x,t), \end{cases} \quad (x,y) \in \partial\Omega, t > 0, \tag{4}$$

where $\psi, \phi, \psi_i, \phi_i (i = 1,2,3,4)$ are known smooth functions.

In the literature, various numerical schemes have been developed for solving partial differential equations [31,33,19,6,8,2,26,27]. One dimensional telegraph equations have been solved using various techniques, namely Taylor matrix method [4], dual reciprocity boundary integral method [2], unconditionally stable finite difference scheme [5], modified B-spline collocation method [6], Chebyshev tau method [7], interpolating scaling function method [1], implicit difference scheme [8], variational iteration method [9], and cubic B-spline collocation method [10]. Two dimensional initial value problem of telegraph equations has been solved using various schemes: Taylor matrix method by Bülbül and Sezer [11] which converts the telegraph equation into the matrix equation, Meshless local weak-strong (MLWS) method and meshless local Petrov-Galerkin (MLPG) method [12], higher order implicit collocation method [13], A polynomial based DQM [14], modified cubic B-spline DQM (MCB-DQM) [33], an unconditionally stable alternating direction implicit scheme [18], A hybrid method due to Dehghan and Salehi [15], compact finite difference scheme by Ding and Zhang [16] with accuracy of order four in both space and time. In [17], Dehghan and Shokri have solved two dimensional telegraph equation with variable coefficients.

Bellman et al. [20,21] developed "differential quadrature (DQ) method" for numerical computation of partial differential equations (PDEs). After seminal work of Bellman et al. and Quan and Chang [22,23], the DQMs have been implemented for various type of base functions, among them, cubic B-spline DQM [24,25], MCB-DQM [31,33,28–30], DQM based on fourier expansion and Harmonic function [34–36], sinc DQM [37], generalized DQM [38], polynomial based DQM [14,40,41], quartic B-spline based DQM [39,42], Quartic and quintic B-spline methods [43], exponential cubic B-spline DQM [44], and extended cubic B-spline DQM [45].

In this paper, we propose a new approach: modified extended cubic-B-spline differential quadrature method (mECDQ) for the numerical computation of two space dimensional second order linear hyperbolic telegraph equation with Dirichlet and Neumann both kind of boundary conditions. The mECDQ method is a new differential quadrature method based on modified extended cubic-B-splines as set of base functions. This converts the initial - boundary value system of the telegraph equation into an initial value system of ODEs, in time. To solve the resulting system of ODEs, we prefer SSP-RK54 [47,48] scheme due to its reduced storage space which results in less accumulation of the numerical errors. The accuracy, efficiency and adaptability of the method are confirmed by taking six test problems of the $2D$ telegraphic equation.

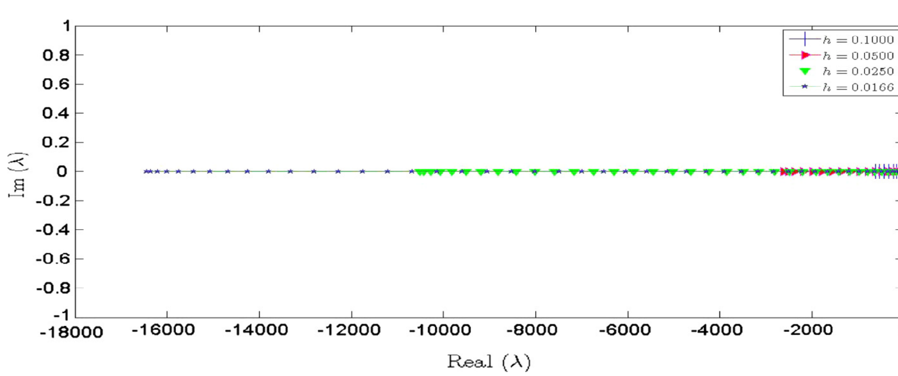

Figure 2 Eigenvalues of B_x and B_y for different grid sizes $h = 0.1, 0.01, 0.025, 0.016$.