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ORIGINAL ARTICLE

New solution algorithm of coupled nonlinear system of Schrodinger equations

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Abstract In this article, new extension of the generalized and improved homogeneous balance method is proposed for constructing new structure of rich class of exact travelling wave solutions of nonlinear evolution equations by using the Maple package. To demonstrate the novelty and motivation of the proposed method, we implement it to the coupled nonlinear system of Schrodinger equations. It is shown that the method provides a powerful mathematical tool for solving nonlinear evolution equations in mathematical physics.

1. Introduction

The investigation of exact solutions of nonlinear equations plays an important role in the study of nonlinear physical phenomena. Searching for explicit solutions of nonlinear evolution equations by using various different methods is the main goal for many researchers, and many powerful methods to construct exact solutions of nonlinear evolution equations have been established and developed such as same works [1–17], the homogeneous balance method [18–19], the F-expansion method [20] and same works [21–26]. One of important methods is (G'/G) – expansion method. The main ideas of the proposed method are that the travelling wave solutions of a nonlinear evolution equation can be expressed by a polynomial in (G'/G) where $G = G(\xi)$ satisfies a second order LODE (see [21]), the degree of the polynomial can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in a given nonlinear evolution equation, and the coefficients of the polynomial can be obtained by solving a set of algebraic equations resulted from the process of using the proposed method. It will be seen that more travelling wave solutions of many nonlinear evolution equations can be obtained by using the (G'/G) – expansion method. It provides the generalized solitary solutions and periodic solutions, as well. Taking advantage of the generalized solitary solutions, we can recover some known solutions obtained by existing methods.

In this paper we extend the homogeneous balance method to a class of nonlinear evolution equations with imaginary number and modulus. We consider the coupled (2 + 1) - dimensional nonlinear system of Schrödinger equations as

$$\begin{cases} iE_t - E_{xx} + E_{yy} + |E|^2E - 2NE = 0, \\ N_{xx} - N_{yy} - (|E|^2)_{xx} = 0, \end{cases} \tag{1}$$

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where $E(x,y,t)$ and $N(x,y,t)$ are complex-valued functions. Nonlinear partial differential equation systems of the type given by (1) play an important role in atomic physics and the functions $E(x,y,t)$ and $N(x,y,t)$ have different physical meanings in different branches of physics. Well-known applications are, for instance, in fluid dynamics and plasma physics. In the context of water waves, $E(x,y,t)$ is the amplitude of a surface wave packet while $N(x,y,t)$ is the velocity potential of the mean flow interacting with the surface waves. However, in the hydrodynamic context, $E(x,y,t)$ is the envelope of the wave packet and $N(x,y,t)$ is the induced mean flow [27–29]. In addition, Eq. (1) are relevant in a number of different physical contexts, describing slow modulation effects of the complex amplitude $N(x,y,t)$, due to a small nonlinearity, on a monochromatic wave in a dispersive medium.

2. Algorithm of the homogeneous balance method

For a given partial differential equation

$$G(u,u_x,u_t,u_{xx},u_{tt},\ldots), \tag{2}$$

our method mainly consists of four steps:

Step 1. We seek complex solutions of Eq. (2) as the following form:

$$u=u(\xi),\quad \xi=ik(x-ct), \tag{3}$$

where k and c are real constants. Under the transformation (3), Eq. (2) becomes an ordinary differential equation

$$N(u,iku',-ikcu',-k^2u'',\ldots), \tag{4}$$

where $u'=\frac{du}{d\xi}$.

Step 2. We assume that the solution of Eq. (4) is of the form

$$u(\xi)=\sum_{i=0}^{n}a_i\phi^i(\xi), \tag{5}$$

where $a_i(i=1,2,\ldots,n)$ are real constants to be determined later and ϕ satisfy the Riccati equation

$$\phi'=a\phi^2+b\phi+c \tag{6}$$

Eq. (6) admits the following solutions:

Case 1: Let $\phi=\sum_{i=0}^{n}b_i\tanh^i\xi$, Balancing ϕ' with ϕ^2 in Eq. (6) gives $m=1$ so

$$\phi=b_0+b_1\tanh\xi, \tag{7}$$

Substituting Eq. (7) into Eq. (6), we obtain the following solution of Eq. (6)

$$\phi=-\frac{1}{2a}(b+2\tanh\xi),\quad ac=\frac{b^2}{4}-1. \tag{8}$$

Case 2: When $a=1, b=0$, the Riccati Eq. (6) has the following solutions

$$\begin{aligned}&\phi=-\sqrt{-c}\tanh\left(\sqrt{-c}\xi\right),\quad c<0\\ &\phi=-\frac{1}{\xi},\quad c<0\\ &\phi=\sqrt{c}\tan\left(\sqrt{c}\xi\right),\quad c>0.\end{aligned} \tag{9}$$

Case 3: We suppose that the Riccati Eq. (6) has the following solutions of the form:

$$\phi=A_0+\sum_{i=1}^{n}\sinh^{i-1}(A_i\sinh\omega+B_i\cosh\omega), \tag{10}$$

where $\frac{d\omega}{d\xi}=\sinh\omega$ or $\frac{d\omega}{d\xi}=\cosh\omega$. It is easy to find that $m=1$ by balancing ϕ' with ϕ^2. So we choose

$$\phi=A_0+A_1\sinh\omega+B_1\cosh\omega, \tag{11}$$

where $\frac{d\omega}{d\xi}=\sinh\omega$, we substitute (11) and $\frac{d\omega}{d\xi}=\sinh\omega$, into (6) and set the coefficients of $\sinh^i\omega,\cosh^i\omega(i=0,1,2;j0,1)$ to zero. We obtain a set of algebraic equations and solving these equations we have the following solutions

$$A_0=-\frac{b}{2a},\quad A_1=0,\quad B_1=\frac{1}{2a} \tag{12}$$

where $c=\frac{b^2-4}{4a}$ and

$$A_0=-\frac{b}{2a},\quad A_1=\pm\sqrt{\frac{1}{2a}},\quad B_1=\frac{1}{2a} \tag{13}$$

where $c=\frac{b^2-1}{4a}$. To $\frac{d\omega}{d\xi}=\sinh\omega$ we have

$$\sinh\omega=-\operatorname{csc}h\xi,\quad \cosh\omega=-\coth\xi \tag{14}$$

from (12)–(14), we obtain

$$\phi=-\frac{b+2\coth\xi}{2a} \tag{15}$$

where $c=\frac{b^2-4}{4a}$ and

$$\phi=-\frac{b\pm\operatorname{csc}h\xi+\coth\xi}{2a} \tag{16}$$

where $c=\frac{b^2-1}{4a}$.

Step 3. Substituting (7)–(16) into (4) along with (6), then the left hand side of Eq. (4) is converted into a polynomial in $F(\xi)$; equating each coefficient of the polynomial to zero yields a set of algebraic equations.

Step 4. Solving the algebraic equations obtained in step 3, and substituting the results into (5), then we obtain the exact travelling wave solutions for Eq. (2).

Remark 1. If $c=0$, then the Riccati Eq. (6) reduces to the Bernoulli equation

$$\phi'=a\phi^2+b\phi, \tag{17}$$

The solution of the Bernoulli Eq. (17) can be written in the following form [23]:

$$\phi=b\left[\frac{\cosh[b(\xi+\xi_0)]+\sinh[b(\xi+\xi_0)]}{1-a\cosh[b(\xi+\xi_0)]-a\sinh[b(\xi+\xi_0)]}\right], \tag{18}$$

where ξ_0 is integration constant.

Remark 2. If $b=0$, then the Riccati Eq. (6) reduces to the Riccati equation

$$\phi'=a\phi^2+c$$

the equation above is the special case of the Riccati Eq. (6).

Remark 3. Also, the Riccati Eq. (6) admits the following exact solution [23]:

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