ARTICLE IN PRESS
Alexandria Engineering Journal (2016) xxx, xxx–xxx

HOSTED BY
ELSEVIER

Alexandria University
Alexandria Engineering Journal
www.elsevier.com/locate/aej
www.sciencedirect.com

ORIGINAL ARTICLE

Particulate suspension slip flow induced by peristaltic waves in a rectangular duct: Effect of lateral walls

Safia Akram [a,*], **Kh.S. Mekheimer** [b,c], **Y. Abd Elmaboud** [d,e]

[a] *Department of Basic Sciences, MCS, National University of Sciences and Technology, Islamabad 4400, Pakistan*
[b] *Mathematics Department, Faculty of Science, Taif University Hawia, P.O. Box 888, Taif, Saudi Arabia*
[c] *Mathematics Department, Faculty of Science, Al-Azhar University, Nasr City, 11884 Cairo, Egypt*
[d] *Mathematics Department, Faculty of Science and Arts, Khulais, King Abdulaziz University (KAU), Saudi Arabia*
[e] *Mathematics Department, Faculty of Science, Al-Azhar University(Assiut Branch), Assiut, Egypt*

Received 29 March 2015; revised 19 September 2016; accepted 20 September 2016

KEYWORDS

Fluid suspension;
Lateral walls;
Peristaltic pumping;
Partial slip

Abstract This paper looks at the influence of lateral walls on peristaltic transport of a particle fluid suspension model applied in a non-uniform rectangular duct with slip boundaries. The peristaltic waves propagate on the horizontal sidewalls of a rectangular duct. The flow analysis has been developed for low Reynolds number and long wavelength approximation. Exact solutions have been established for the axial velocity and stream function. The effects of aspect the ratio β (ratio of height to width) and the volume fraction density of the particles C on the pumping characteristics are discussed in detail. The expressions for the pressure rise and friction forces on the wall of a rectangular duct were computed numerically and were plotted with variation of the flow rate for different values of the parameters. It is observed that in the peristaltic pumping ($\Delta p > 0, Q > 0$) and retrograde pumping ($\Delta p > 0, Q < 0$) regions the pumping rate increases with an increase in M, while in the copumping region ($\Delta p < 0, Q > 0$) the behavior is quite opposite. Furthermore it is also observed that the pressure rise increases in the upper half of the channel and decreases in the lower half of the channel with the increase in l_{slip} parameter.

1. Introduction

In the recent years peristaltic transport becomes an important subject for the researcher in biofluid. The peristaltic transport is very important because, it is a primary transport mechanism inherent in many tubular organs of the human body such as

* Corresponding author.
E-mail addresses: drsafiaakram@gmail.com, safia_akram@yahoo.com (S. Akram).
Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2016.09.011
1110-0168 © 2016 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V.

the gastrointestinal tract, the urethra, the ureter and arterioles. The mechanism of peristaltic transport has been employed for industrial applications such as sanitary fluid transport, blood pumps in heart lung machine and transport of corrosive fluids. Peristaltic flow is generated in a channel (or a circular tube) when a progressive wave travels along the wall. Many studies have been carried out for understanding the characteristics of the transport mechanism associated with peristaltic flow under the assumption of low Reynolds number and infinitely long wavelength such as Jaffrin and Shapiro [1] explained the basic principles of peristaltic pumping and brought out clearly the significance of the various parameters governing the flow. A number of analytical, numerical and experimental recent studies of peristaltic flows of different fluids have been reported [2–13].

The theory of the two phase fluid is very useful in understanding a number of diverse physical problems including powder technology, fluidization. The continuum theory of mixtures is applicable to hydrodynamics of biological systems, because it provides an improved understanding of diverse subjects such as diffusion of proteins, the rheology of blood, swimming of micro-organisms and particle deposition. The model of a particulate suspension with peristaltic transport is investigated by many authors [14–18]. There are a recent few studies take into account the effect of the sidewalls in peristaltic transport such as Reddy et al. [19], and Nadeem and Akram [20]. None of the previous studies take into consideration slip boundaries and the sidewall effects especially in the non-uniform channel when the fluid model is a two phase fluid. So our motivation was to study the influence of lateral walls on peristaltic transport of a particle fluid suspension model applied in a non-uniform rectangular duct with slip boundaries.

2. Mathematical formulation

Let us consider the two-phase (fluid particles) flow in a non-uniform duct of rectangular cross section. The duct walls are flexible, and an infinite train of sinusoidal waves propagate with constant velocity c along the walls parallel to the XY plane in the axial direction. Cartesian coordinate system (X, Y, Z) is with X, Y, and Z axes corresponding to axial, lateral, and vertical directions, respectively, of a rectangular duct. We assume that there is no flow in the lateral direction. So, the velocity vector in this direction will be zero. The sinusoidal waves propagating along the channel walls are of the following forms:

$$Z' = H'(X', t') = \pm a \pm kx \pm b \sin\left[\frac{2\pi}{\lambda}(X' - ct')\right], \quad (1)$$

where a is the half width of the channel, b is amplitude of the wave, λ is the wavelength, d is the channel width, $k(\ll 1)$ is a constant whose magnitude depends on the length of the channel, c is the velocity of propagation, and t' is the time. The equations governing the conservation of mass and linear momentum for both the fluid and particle phases using a continuum approach are expressed as follows:

Fluid phase

$$\frac{\partial}{\partial X'}((1-C)U'_f) + \frac{\partial}{\partial Z'}((1-C)V'_f) = 0, \quad (2)$$

$$(1-C)\rho_f\left(\frac{\partial U'_f}{\partial t'} + U'_f\frac{\partial U'_f}{\partial X'} + V'_f\frac{\partial U'_f}{\partial Z'}\right) = -(1-C)\frac{\partial P'}{\partial X'} + (1-C)\mu_s(C)\nabla^2 U'_f + CS\left(U'_P - U'_f\right), \quad (3)$$

$$(1-C)\rho_f\left(\frac{\partial V'_f}{\partial t'} + U'_f\frac{\partial V'_f}{\partial X'} + V'_f\frac{\partial V'_f}{\partial Z'}\right) = -(1-C)\frac{\partial P'}{\partial Z'} + (1-C)\mu_s(C)\nabla^2 V'_f + CS\left(V'_P - V'_f\right), \quad (4)$$

Particulate phase

$$\frac{\partial}{\partial X'}(CU'_P) + \frac{\partial}{\partial Z'}(CV'_P) = 0, \quad (5)$$

$$C\rho_P\left(\frac{\partial U'_P}{\partial t'} + U'_P\frac{\partial U'_P}{\partial X'} + V'_P\frac{\partial U'_P}{\partial Z'}\right) = -C\frac{\partial P'}{\partial X'} + CS\left(U'_f - U'_P\right), \quad (6)$$

$$C\rho_P\left(\frac{\partial V'_P}{\partial t'} + U'_P\frac{\partial V'_P}{\partial X'} + V'_P\frac{\partial V'_P}{\partial Z'}\right) = -C\frac{\partial P'}{\partial Z'} + CS\left(V'_f - V'_P\right), \quad (7)$$

where $\left(U'_f, V'_f\right)$ and $\left(U'_P, V'_P\right)$ are the velocity components of the fluid and the particle phase respectively, P' is the pressure, C is the volume fraction density of the particles [17], $\mu_s(C)$ is the mixture viscosity (effective or apparent viscosity of suspension) and S is the drag coefficient of interaction for the force exerted by one phase on the other. The expression for the drag coefficient of interaction, S and the empirical relation for the velocity of the suspension, μ_s for the present problem is selected as done by Srivastava and Saxena [17]

$$S = \frac{9}{2}\frac{\mu_0}{a_0^2}\lambda'(C),$$

$$\lambda'(C) = \frac{4 + [8C - 3C^2]^{\frac{1}{2}} + 3C}{(2-3C)^2}, \quad (8)$$

$$\mu_s = \mu_s(C) = \frac{\mu_0}{1 - mC},$$

$$m = 0.070\exp\left[2.49C + \frac{1107}{T}\exp(-1.69C)\right], \quad (9)$$

where a_0 is the radius of each solid particle suspended in the fluid, μ_0 is the constant fluid viscosity and T is measured in absolute temperature (K^0). The formula (9) has been tested by Charm and Kurland [21] by using a cone and a plate viscometer, and it has been proclaimed that it is reasonably accurate up to $C = 0.6$.

Let us define a wave frame (x', y', z') moving with the velocity c away from the fixed frame (X', Y', Z') by the transformation

$$x' = X' - ct', y' = Y', \quad z' = Z', \quad u'_{f,p} = U'_{f,p} - c,$$
$$v'_{f,p} = V'_{f,p}, \quad p'(x, z) = P'(X, Z, t). \quad (10)$$

where $u'_{f,p}$ and $v'_{f,p}$ are the velocity components of fluid and particle respectively, p' and P' are the pressures in wave and fixed