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ORIGINAL ARTICLE

An approximate solution for the MHD nano boundary-layer flows over stretching surfaces in a porous medium by rational Legendre collocation method

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Abstract Solutions for the magnetohydrodynamic nano boundary layer fluid flow over a permeable stretching surface embedded in a porous medium are obtained numerically by rational Legendre collocation method. Two-dimensional and axisymmetric flows induced by stretching of the surface are considered. The effects of magnetic parameter, the porous parameter, first and second order slip and suction/injection parameters on the flow are discussed. A comparison of numerical results with previous published results is made and the results are found to be in good agreement.

1. Introduction

Boundary layer flow over a stretching sheet has many practical applications in numerous conventional industries as well as in micro- and nano-technologies. Extrusion processes in metallurgy, manufacturing of glass sheets, paper and textile production, micro-pumps, micro-valves and micro-nozzles are some applications [1–3]. Early work on this area [4,5] applied the common no-slip condition at the surface. For non-Newtonian liquids or when the flow is at the micro- and nano-scale, the no-slip condition at surface is not accurate and partial slip has to be allowed [6–8]. If the stretching sheet is permeable the no-penetration condition has to be replaced with a suction/injection condition. In many applications, momentum and heat transfers in the boundary layer flow over a stretching/shrinking sheet are controlled by a magnetic field. Almost in all cases, the usual approach for solving the relevant boundary value problem uses a similarity transform to convert the governing Navier-Stokes equations into a third order nonlinear ordinary differential equation. The converted boundary value problem has been studied analytically and numerically by many authors. In addition to those mentioned above, the works of Andersson [9], Wang [10], Van Gorder et al. [11], Arial [12] and Liao [13] can be mentioned. Existence and uniqueness results were presented by Wang [10] and Akyildiz

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Table 1 Comparison of the approximate shear stress at surface $f''(0)$ with exact values, for 2–D flow and for various values of K_p, s, γ, δ and M.

K_p	s	γ	δ	M	Approximate	Exact [3]
0.5	−3	0	0	0	−0.79128784747	−0.7912878475
0.5	−3	0	0	2	−1.19258240356	−1.1925824036
0.5	−3	0	0	5	−1.70156211871	−1.7015621187
0.5	−2	0	0	0	−0.99999999999	−1.0000000000
0.5	−2	0	0	2	−1.44948974278	−1.4494897428
0.5	−2	0	0	5	−1.9999999999	−2.0000000000
100	−3	0	0	0	−0.30554700852	−0.3055470085
100	−3	0	0	2	−0.79346898823	−0.7934689882
100	−3	0	0	5	−1.37402157263	−1.3740215726
100	−2	0	0	0	−0.41774468793	−0.4177446879
100	−2	0	0	2	−1.00249843945	−1.0024984395
100	−2	0	0	5	−1.64764045897	−1.6476404590
∞	2	0	0	0	−2.41421359238	−2.4142135624
∞	2	0	0	2	−3.00000000000	−3.0000000000
∞	2	0	0	5	−3.64575131106	−3.6457513111
∞	2	0	1	0	−0.68232780665	−0.6823278038
∞	2	0	1	2	−0.73727772070	−0.7372777207
∞	2	0	1	5	−0.77750548587	−0.7775054859
∞	2	0	−1	0	−0.38942825653	−0.38942825653
∞	2	0	−1	2	−0.31981873553	−0.3198187355
∞	2	0	−1	5	−0.26640223717	−0.2664022372
∞	2	0	−3	0	−0.1515009701123	−0.15150097112
∞	2	0	−3	2	−0.11631788389	−0.1163178839
∞	2	0	−3	5	−0.093856888950	−0.0938568890
∞	2	1	−3	0	−0.131788668953	−0.13178866911
∞	2	1	−3	2	−0.10423697133	−0.1042369713
∞	2	1	−3	5	−0.08581379397	−0.0858137940
∞	2	1	−5	0	−0.0862046926142	−0.08620469270
∞	2	1	−5	2	−0.0663998888764	−0.0663998889
∞	2	1	−5	5	−0.05389963358	−0.0538996336

et al. [7]. The flow fields considered so far have been two-dimensional and axisymmetric. In what follows we focus on some recent research that is pertinent to the subject of this paper. Fang et al. [14] found analytic solutions of Magnetohydrodynamic (MHD) viscous flow over a stretching sheet. Later the solution is obtained for a shrinking sheet with a second-order slip model [15]. Abbasbandy and Ghehsareh [16] used the Hankel-Pade method to solve MHD flow over a nonlinear stretching sheet and nano boundary layer over stretching surfaces. Turkyilmazoglu [2] analytically investigated the effect of various physical properties for the flow and heat transfer over stretching/shrinking sheet. Aly and Hassan [17] have analytically and numerically studied the MHD nano boundary layer over a stretching surface through a porous medium with a first order slip condition. The same problem was considered by Aly and Vajravelu [3] with a second order slip condition where the exact solution is obtained in the two-dimensional case and in the axisymmetric case the Chebyshev pseudospectral differentiation matrix technique was used to find the solution numerically. Spectral collocation methods are used for numerical solution of nonlinear differential equations. Khader [18] applied Laguerre collocation method for investigating the flow and heat transfer due to a permeable stretching surface with second-order slip and viscous dissipation. Recently, an efficient and powerful class of spectral methods based on the rational orthogonal basis functions has been introduced and developed for solving boundary value problems on unbounded intervals [19–21]. In [22], authors have introduced orthogonal rational Legendre basis functions and developed a rational Legendre spectral method to solve the Korteweg-de Vries equation on a semi-infinite interval. Newly, several types of the rational spectral techniques have been developed and successfully employed to deal with various types of boundary layer flow problems [23–26]. In the present study a collocation method based on the rational Legendre polynomials is used to solve the nonlinear boundary value problem for the MHD nano boundary-layer flow over a stretching surface in a porous medium. The effects of various parameters on the flow are presented. The problem is formulated in Section 2. Rational Legendre polynomials and rational Legendre collocation method are introduced in Sections 3 and 4. The numerical implementation is presented in Section 4. Section 5 contains results and discussions. Finally, the concluding remarks are given in Section 6.