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ORIGINAL ARTICLE

Analysis of convective longitudinal fin with temperature-dependent thermal conductivity and internal heat generation

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KEYWORDS

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Abstract In this study, analysis of heat transfer in a longitudinal rectangular fin with temperature-dependent thermal conductivity and internal heat generation was carried out using finite difference method. The developed systems of non-linear equations that resulted from the discretization using finite difference scheme were solved with the aid of MATLAB using ***fsolve***. The numerical solution was validated with the exact solution for the linear problem. The developed heat transfer models were used to investigate the effects of thermo-geometric parameters, coefficient of heat transfer and thermal conductivity (non-linear) parameters on the temperature distribution, heat transfer and thermal performance of the longitudinal rectangular fin. From the results, it shows that the fin temperature distribution, the total heat transfer, and the fin efficiency are significantly affected by the thermo-geometric parameters of the fin. Also, for the solution to be thermally stable, the fin thermo-geometric parameter must not exceed a specific value. However, it was established that the increase in temperature-dependent properties and internal heat generation values increases the thermal stability range of the thermo-geometric parameter. The results obtained in this analysis serve as basis for comparison of any other method of analysis of the problem.

1. Introduction

The increasing demand and the quest for high-performance heat transfer components with progressively smaller weights, volume, costs or accommodating shapes have greatly increased the use of extended surfaces to enhance heat dissipation from hot primary surfaces. In the design and construction of various types of heat-transfer equipment and components such as air conditioner, refrigerator, superheaters, automobile, power plants, heat exchangers, convectional furnaces, economizers, gas turbines, chemical processing equipment, oil carrying pipelines, computer processors, electrical chips, etc., extended surfaces are used to implement the flow of heat between a source and a sink. In practice, various types of fins with different geometries are used, but due to the simplicity of its design and ease of construction and manufacturing process, the rectangular fins are widely applied in heat-transfer equipment. Also, for ordinary fins problem, the thermal properties of the fin and the surrounding medium (thermal conductivity

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Nomenclature

a_r	aspect ratio or extension factor
A	cross sectional area of the fins, m^2
Bi	Biot number
h	heat transfer coefficient, $W\,m^{-2}\,k^{-1}$
h_b	heat transfer coefficient at the base of the fin, $W\,m^{-2}\,k^{-1}$
H	dimensionless heat transfer coefficient at the base of the fin, $W\,m^{-2}\,k^{-1}$
j	geometric parameter
k	thermal conductivity of the fin material, $W\,m^{-1}$ -k^{-1}
k_b	thermal conductivity of the fin material at the base of the fin, $W\,m^{-1}\,k^{-1}$
K	dimensionless thermal conductivity of the fin material, $W\,m^{-1}\,k^{-1}$
L	Length of the fin, m
M	dimensionless thermo-geometric fin parameter
m^2	thermo-geometric fin parameter m^{-1}
n	convective heat transfer power
P	perimeter of the fin, m
T	Temperature, K
T_∞	ambient temperature, K
T_b	Temperature at the base of the fin, K
x	fin axial distance, m
X	dimensionless length of the fin
Q	dimensionless heat transfer
q_i	the uniform internal heat generation in W/m^3
Greek symbols	
β	thermal conductivity parameter or non-linear parameter
δ	thickness of the fin, m
δ_b	fin thickness at its base
γ	dimensionless internal heat generation parameter
θ	dimensionless temperature
θ_b	dimensionless temperature at the base of the fin
η	efficiency of the fin
ε	effectiveness of the fin

and heat transfer coefficient) are assumed to be constant, but if a large temperature difference exists within the fin, typically, between the tip and the base of the fin, the thermal conductivity and the heat transfer coefficient are not constant but temperature-dependent. Therefore, while analyzing the fin, the effects of temperature-dependent thermal properties must be taken into consideration. In carrying out such an analysis, the thermal conductivity may be modeled for such and other many engineering applications by power law and by linear dependency on temperature, while the heat transfer coefficient can be expressed as power law for which the exponents represent different phenomena as reported by Khani and Aziz [1], Ndlovu and Moitsheki [2]. Such dependency of thermal conductivity and heat transfer coefficient on temperature renders the problem highly non-linear and difficult to solve analytically. It is also very realistic to consider the temperature-dependent internal heat generation in the fins as applied in electric-current carrying conductor, nuclear rods or any other heat generating components of thermal systems.

Over the past few decades, the research on the temperature-dependent thermal conductivity and heat transfer coefficient has been on-going in the literature. Also, the solutions of the highly non-linear differential equations have been constructed using different techniques. Aziz and Enamul-Huq [3] and Aziz [4] applied regular perturbation expansion to study a pure convection fin with temperature dependent thermal conductivity. Few years later, Campo and Spaulding [5] predicted the thermal behavior of uniform circumferential fins using method of successive approximation. Chiu and Chen [6] and Arslanturk [7] adopted the Adomian decomposition method (ADM) to obtain the temperature distribution in a pure convective fin with variable thermal conductivity. The same problem was solved by Ganji [8] with the aid of the homotopy perturbation method originally proposed by He [9]. In the same year, Chowdhury and Hashim [10] applied Adomian decomposition method to evaluate the temperature distribution of straight rectangular fins with temperature-dependent surface flux for all possible types of heat transfer while in the following year, Rajabi [11] applied homotopy perturbation method (HPM) to calculate the efficiency of straight fins with temperature-dependent thermal conductivity. Also, a year later, Mustapha [12] adopted homotopy analysis method (HAM) to find the efficiency of straight fins with temperature-dependent thermal conductivity. Meanwhile, Coskun and Atay [13] utilized the variational iteration method (VIM) for the analysis of convective straight and radial fins with temperature-dependent thermal conductivity. Also, Languri et al. [14] applied both the variation iteration and homotopy perturbation methods for the evaluation of the efficiency of straight fins with temperature-dependent thermal conductivity while Coskun and Atay [15] applied variational iteration method to analyze the efficiency of convective straight fins with temperature-dependent thermal conductivity. Besides, Atay and Coskun [16] employed variation iteration and finite element methods to carry out comparative analysis of power-law-fin type problems. Domairry and Fazeli [17] used homotopy analysis method to determine the efficiency of straight fins with temperature-dependent thermal conductivity. Chowdhury et al. [18] investigated a rectangular fin with power law surface heat flux and made a comparative assessment of results predicted by HAM, HPM, and ADM. Khani et al. [19] used Adomian decomposition method (ADM) to provide series solution to fin problems with a temperature-dependent thermal conductivity while Moitheki et al. [20] applied the Lie symmetry analysis to provide exact solutions of the fin problem with a power-law temperature-dependent thermal conductivity and while Hosseini et al. [21] applied homotopy analysis method to generate approximate but accurate solution of heat transfer in fin with temperature-dependent internal heat generation and thermal conductivity. Sadollah et al. [22]

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