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ORIGINAL ARTICLE

MHD squeezed flow of Carreau-Yasuda fluid over a sensor surface

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Abstract An attempt has been made to examine the two dimensional free stream squeezed flow through a horizontal sensor surface of an electrically conducting Carreau-Yasuda fluid subject to transverse magnetic field. The governing nonlinear partial differential equations are modeled and then simplified with the aid of suitable similarity transformations. Well known numerical scheme Runge–Kutta–Fehlberg method is utilized to solve the system. Concrete graphical analysis is carried out to investigate the behavior of different pertinent parameters on velocity profile with inclusive discussion. Numerical influence of friction factor is also discussed.

1. Introduction

Due to immense industrial and engineering applications, it is supreme interest to examine the magneto-hydrodynamic flow. The main purpose of magneto-hydrodynamic principles is to disturb the flow field in a preferred direction by varying the structure of the boundary layer. Thus, in order to amend the flow kinematics, the idea to implement MHD appears to be more easy and consistent. In medicine and biology, MHD has been receiving emergent attention of physiologists, fluid dynamicists and medical practitioners, due to its significance in biomedical engineering as well as in the cure of various pathological circumstances. Further example that fits to the above class of problems is modern metallurgical and metal-working processes. Mukhopadhyay [1] studied the MHD flow of viscous fluid induced due to exponentially stretching sheet. She obtained the solution of velocity and temperature functions by shooting method. Moreover, she found that as the magnetic field increases the surface shear stress enhances. Jat et al. [2] discussed the heat transfer flow of viscous fluid over an exponentially stretching sheet under the influence of radiation, viscous dissipation and magnetic force. They found that heat transfer rate enriches as the magnetic field increases. Desale et al. [3] carried out an investigation to see the effect of boundary layer MHD flow over a nonlinear stretching sheet. They concluded that the influence of magnetic field in reduction of velocity profile is more prominent than nonlinear stretching parameter. Uddin et al. [4] explored the numerical solution of MHD nonlinear nanomaterial stretching or shrinking sheet along with convective heating and Navier slip conditions. They illustrated that by increasing magnetic field the concentration and temperature profiles enhance. Awais et al. [5] discussed the MHD peristaltic flow of Jeffery fluid in an asymmetric channel with convective boundary conditions. They calculated the solution by using long wavelength approximation. Hayat et al. [6] studied the MHD peristaltic flow of

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nanofluid in a channel with slip, wall properties and Joule heating. They found that both the nanoparticles concentration and temperature are increasing functions of thermophoresis and Brownian motion parameters. Akbar et al. [7] investigated the numerical solution of MHD Eyring-Powell fluid over a stretching sheet and found that the increment in magnetic field and Eyring–Powell fluid parameters reduces the velocity profile. Mabood et al. [8] analyzed the effects of MHD and viscous dissipation over a non-linear stretching sheet. They concluded that local Nusselt and Sherwood numbers reduces by increasing magnetic field. Hakeem et al. [9] examined the numerical solution for the MHD flow of nanofluid towards a stretching sheet with thermal radiation and second order slip conditions. They initiated that the second order slip and magnetic field have imperative influence on both shrinking and stretching sheet. By increasing the magnetic field the lower branch solution of shrinking sheet disappears. Sheikh et al. [10] discussed the heat generation/absorption and thermophoresis effects on MHD flow over an oscillatory stretching sheet. They effectuated that motion of the fluid deaccelerates by increasing magnetic field. Malik et al. [11] deliberated the solution of MHD flow of Williamson fluid towards a stretching cylinder by using shooting method. They found that velocity profile reduces by increasing magnetic field and Williamson parameter. Ali et al. [12] deliberated the numerical and analytical solutions of radially non-linear stretched surface with slip effects. They compared analytic and numerical results of the problem. Mahanthesh et al. [13] discussed the MHD flow of nanofluid (Cu, Al_2O_3 and TiO_2) over a bidirectional non-linear stretching surface. They calculated the results for both non-linear and linear stretching sheet cases. Some other articles related to MHD flow are described in Refs. [14–23].

Researchers currently have concentrated considerably on sensor surface due to its importance in biological and chemical processes. Microcantilever is such an example which has the ability to detect various diseases or can be used to accurately sense many bio-warfare or hazardous agents. Khaled and Vafai [24] studied the role of sensor surface confined inside a squeezed channel. They inaugurated that by increasing the wall suction velocity and magnetic field the local Nusselt number enriches. Rashidi et al. [25] obtained approximate solutions for two-dimensional unsteady squeezing flow between two parallel plates. They concluded that homotopy analysis method provides good approximation for both strongly and weakly non-linear problems. Haq et al. [26] analyzed the hydromagnetic squeezed flow of nanofluid towards a sensor surface. They initiated that by raising the nanoparticle volume fraction the velocity profile reduces.

Most of the fluids used in industries are non-Newtonian [27–31]. In such fluids there is no linear relationship between stress and deformation rate. Examples of such fluids are pulps, molten polymers, animal blood, etc. Carreau-Yasuda fluid is such a model that predicts the shear thinning/thickening behavior. Hayat et al. [32] discussed the peristaltic flow of Carreau-Yasuda Fluid with slip effects in a curved channel. They found that increase in velocity slip parameter decreases the retrograde pumping and peristaltic regions. The peristaltic flow of Carreau–Yasuda fluid model in an asymmetric channel with Hall and Ohmic heating effects was studied by Hayat et al. [33]. They made a comparative study for viscous, Carreau, and Carreau-Yasuda fluids. Hayat et al. [34] discussed the mixed convective peristaltic transport of Carreau–Yasuda fluid with chemical reaction and thermal deposition. They studied the problem by assuming the large wavelength and small Reynolds number approximation.

The above literature review illustrates that no one has studied so far the unsteady MHD boundary layer flow of Carreau-Yasuda fluid over a horizontal sensor surface inward a squeezing horizontal channel. After employing the appropriate similarity transformations the required equation of motion is reduced into non-linear differential form. The numerical solution of the resulting equation is calculated through Runge-Kutta- Fehlberg method. Finally, the impact of dimensionless parameters on the flow streamlines and velocity profile along with the friction factor are discussed with the help of graphs and table.

2. Mathematical formulation

Consider unsteady two-dimensional Carreau-Yasuda fluid flow between two infinite parallel plates. The microcantilever sensor of length L is enclosed inside the channel and the upper surface of the channel is squeezed, but the lower surface is fixed. It is assumed that the height $h(t)$ is greater than the boundary layer thickness. The flow is driven by the external free stream velocity $U(x,t)$ and the magnetic field of strength B_m is applied normal to the channel as shown in Fig. 1.

The constitutive equation of Carreau-Yasuda fluid is

$$\boldsymbol{\tau} = \left[\mu_\infty + (\mu_0 - \mu_\infty)\left(1 + (\Gamma\dot{\gamma})^d\right)^{\frac{n-1}{d}}\right]\mathbf{A}_1, \tag{1}$$

in which, τ is the extra stress tensor, A_1 is the first Rivlin-Ericksen tensor, μ_0 is the zero shear rate viscosity, μ_∞ is the infinite shear rate, d, n and Γ are the Carreau-Yasuda fluid parameters and $\dot{\gamma}$ is defined as $\dot{\gamma} = \sqrt{2tr(D^2)}$, where $D = \frac{1}{2}[gradV + (gradV)^T]$. Here it is assumed that $\mu_\infty = 0$, then Eq. (1) takes the following form:

$$\boldsymbol{\tau} = \boldsymbol{\mu}_0[1 + (\Gamma\dot{\gamma})^d]^{\frac{n-1}{d}}\mathbf{A}_1, \tag{2}$$

According to the present situation following are the equations of continuity, momentum and free stream:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\frac{\partial^2 u}{\partial y^2} + \frac{(n-1)v}{d}\Gamma^d(d+1) \times \left(\frac{\partial u}{\partial y}\right)^d\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_m B_m^2}{\rho}u, \tag{4}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\sigma_m B_m^2}{\rho}U, \tag{5}$$

where v is the kinematic viscosity, p is the fluid pressure, u and v are the x-component and y-components of velocity respectively, ρ is the density, n is the power law index, σ_m is the electric charge density, d is the generalized parameter and Γ is the time constant.

Eq. (4) is valid within the boundary layer while Eq. (5) is the outer stream flow which is presumed to be inviscid. Therefore, this assumption is devalued when the sensor length is taken to be much closer to the channel's height. Momentum