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ORIGINAL ARTICLE

A numerical study of unsteady non-Newtonian Powell-Eyring nanofluid flow over a shrinking sheet with heat generation and thermal radiation

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Abstract In this paper we investigate the unsteady boundary-layer flow of an incompressible Powell-Eyring nanofluid over a shrinking surface. The effects of heat generation and thermal radiation on the fluid flow are taken into account. Numerical solutions of the nonlinear differential equations that describe the transport processes are obtained using a multi-domain bivariate spectral quasilinearization method. This innovative technique involves coupling bivariate Lagrange interpolation with quasilinearization. The solutions of the resulting system of equations are then obtained in a piecewise manner in a sequence of multiple intervals using the Chebyshev spectral collocation method. A parametric study shows how various parameters influence the flow and heat transfer processes. The validation of the results, and the method used here, has been achieved through a comparison of the current results with previously published results for selected parameter values. In general, an excellent agreement is observed. The results from this study show that the fluid parameters ε and δ reduce the flow velocity and the momentum boundary-layer thickness. The heat generation and thermal radiation parameters are found to enhance both the temperature and thermal boundary-layer thicknesses.

1. Introduction

The study of flow and transport processes in non-Newtonian fluids has gained much research attention in recent years due to the important use of various such fluids in industry, biological processes and chemical engineering. A few examples of such applications include in the manufacture of optical fibers and plastic polymers, clay coating and in cosmetic products. Due to the wide diversity of non-Newtonian fluids, the important rheological characteristics of such flows cannot be addressed by a single constitutive relation between the shear stress and the shear rate. Significant contributions to the study of non-Newtonian fluid models with a variety of rheological

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properties have been made by Harris [1] and Bird et al. [2]. An interesting non-Newtonian fluid is the Powell-Eyring fluid, which, although very complex, has certain advantages over other non-Newtonian fluid models, Powell and Eyring [3] in some respects. These include the fact that the model is derived from kinetic theory of liquids instead of the empirical relation, and that the Powell-Eyring fluid model reduces to the Newtonian fluid for low and high shear rates. A common example of an Powell-Eyring fluid is human blood. Due to the importance of Powell-Eyring fluids, many researchers have studied different physical properties of Powell-Eyring fluids. These include the study of Malik et al. [4] who investigated mixed convection in an MHD Eyring-Powell nanofluid over a stretching sheet. They showed that the fluid was accelerated by increasing the Eyring-Powell parameter and the mixed convection parameter. Hayat et al. [5] investigated radiation effects on the flow of a Powell-Eyring fluid past an unsteady inclined stretching sheet with a non-uniform heat source/sink. They showed that the velocity and temperature profiles generally decrease with the unsteadiness parameter. An increase in the radiation parameter was shown to increase the heat flux from the plate, which in turn enhanced the fluid velocity and temperature.

The unsteady incompressible Eyring-Powell flow in a pipe with porous walls was investigated by Zaman et al. [6] using the homotopy analysis method. Series of solutions of an unsteady Eyring Powell nanofluid flow about a rotating cone were obtained by Nadeem and Saleem [7]. In their investigation, they observed that the nano particle volume fraction decreased with the particle Brownian motion and the Lewis number. Jalil et al. [8] found self-similar solutions for flow and heat transfer in an Powell-Eyring fluid flow over a moving surface with a variable surface temperature. Roşca and Pop [9] studied the boundary-layer flow and heat transfer in a Powell-Eyring fluid over a shrinking surface. In their study, numerical results were obtained using the Matlab inbuilt function bvp4c. They found dual solutions for negative values of the stretching parameter and stability analysis showed that the first (upper branch) solution was stable and physically realizable, while the second (lower branch) solution is not stable and, therefore, not physically possible. Other Powell-Eyring studies were carried out by Hayat et al. [10,11], Asmat et al. [12], Khan and Sultan [13], Nadeem and Saleem [14].

In the past few years, the study of the flow, and the thermophysical properties of nanofluids has become a topic of major interest due to the huge potential for the use of these fluids as efficient heat transfer fluids, and in some biomedical applications. The concept of a nanofluid was first proposed by Chol [15] when he showed that by adding a small quantity of nanoparticles to conventional heat transfer liquids, the thermal conductivity of the fluid improved by approximately a factor of two. A non-homogeneous two component equation for nanofluids was developed by Buongiorno [16]. He introduced seven slip mechanisms between nanoparticles and the base fluid. He took into account particle Brownian motion and thermophoresis and showed that Brownian motion and thermophoresis have significant influence on forced convection in nanofluids. Rohni et al. [17] used the shooting method to find a numerical solution of the equations for an unsteady shrinking surface with wall mass suction using the nanofluid model proposed by Buongiorno [16]. Zaimi et al. [18] used the Buongiorno model to investigate unsteady flow due to a contracting cylinder. The equations were solved using the shooting method. They obtained dual solutions for a certain range of the unsteadiness parameter and also observed that the skin friction coefficient, the Nusselt number and the Sherwood number decreased with increasing values of the unsteadiness parameter. Multiple solutions of MHD boundary layer flow and heat transfer behavior of nanofluids induced by a power-law stretching/shrinking permeable sheet with viscous dissipation were presented by Dhanai et al. [19] using the shooting method. They showed the existence of dual solutions for different flow parameters. Further, they found that viscous dissipation is important whereas the Brownian motion has negligible effect on the rate of heat transfer. Recently, Haroun et al. [20] used the spectral relaxation method to solve the equations that model the unsteady MHD mixed convection in a nanofluid due to a stretching or shrinking surface with suctionand/or injection. Their results showed that the skin friction factor increases with both an increase in the nanoparticle volume fraction and the stretching rate, and that an increase in the nanoparticle volume fraction leads to a reduction in the wall mass transfer rate. Numerical solutions of heat and mass transfer of nanofluid through an impulsively stretching vertical surface were presented by Haroun et al. [21]. Other recent studies of nanofluid flows include those by Haroun et al. [22], Dalir et al. [23], Abolbashari et al. [24], Heidary et al. [25], Mansur et al. [26], Haq et al. [27], Mehmood et al. [28], Sher Akbar et al. [29–32].

The study of unsteady Powell-Eyring Nanofluid has not been given much attention so far. The aim of this study was to investigate the flow of an unsteady Powell-Eyring nanofluid over a shrinking sheet with heat generation and thermal radiation effects. The traditional model of Jalil et al. [8] and Rosca and Pop [9] is revised to incorporate the effects of thermal radiation, heat generation, thermophoresis and Brownian motion. The equations are solved numerically using a multi-domain or multi-stage bivariate spectral quasilinearization method (MD-BSQLM). Examples of multi-interval methods that have been developed to solve IVPs include the piecewise spectral homotopy analysis [33,34], the piecewise homotopy perturbation method [35], the multi-stage differential transformation method [36,37], multistage Adomian decomposition method [38,39], the multi-stage quasilinearization method [40,41], and multistage spectral relaxation method [42,43]. The MD-BSQLM is a novel technique that has not been used to solve systems of nonlinear partial differential equations. In this investigation, we extend the use of the method to systems of nonlinear partial differential equations. The multi-domain bivariate spectral quasilinearization method is based on linearizing the governing nonlinear system of PDEs using the Newton–Raphson based quasilinearization method of Bellman and Kalaba [47] and then integrating the resulting equation in multiple sub-intervals using the Chebyshev spectral collocation method with Lagrange interpolation polynomials as basis functions. The Chebyshev spectral collocation method with the Lagrange interpolation polynomials is applied on the linearized nonlinear systems of partial differential equations independently in both space and time direction. These useful features of the MD-BSQLM enable the approach to yield a very accurate solution and lead to significant computational time saving. The approach has a much better region of convergence for the approximate solution when compared to other Chebyshev spectral collocation based methods such as bivariate spectral homotopy analysis method [44], bivariate

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