ARTICLE IN PRESS

Alexandria Engineering Journal (2016) xxx, xxx–xxx

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Alexandria Engineering Journal

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ORIGINAL ARTICLE

MHD oscillatory flow through a porous channel saturated with porous medium

J.A. Falade [a], Joel C. Ukaegbu [b], A.C. Egere [b], Samuel O. Adesanya [b,*]

[a] Department of Physical Sciences, Redeemer's University, Nigeria
[b] Department of Mathematical Sciences, Redeemer's University, Nigeria

Received 6 January 2016; revised 19 September 2016; accepted 24 September 2016

KEYWORDS

Oscillatory flow;
Porous medium;
Magnetic field;
Fluid slip;
Suction/injection

Abstract In this paper, we investigate the effect of suction/injection on the unsteady oscillatory flow through a vertical channel with non-uniform wall temperature. The fluid is subjected to a transverse magnetic field and the velocity slip at the lower plate is taken into consideration. Exact solutions of the dimensionless equations governing the fluid flow are obtained and the effects of the flow parameters on temperature, velocity profiles, skin friction and rate of heat transfer are discussed and shown graphically. It is interesting to note that skin friction increases on both channel plates as injection increases on the heated plate.

1. Introduction

The study of oscillatory flow of an electrically conducting fluid through a porous channel saturated with porous medium is important in many physiological flows and engineering applications such as magneto-hydrodynamic (MHD) generators, arterial blood flow, petroleum engineering and many more.

Several authors have studied the flow and heat transfer in oscillatory fluid problems. To mention just a few, Makinde and Mhone [1] investigated the forced convective MHD oscillatory fluid flow through a channel filled with porous medium, and analyses were based on the assumption that the plates are impervious. In a related study, Mehmood and Ali [2] investigated the effect of slip on the free convective oscillatory flow through vertical channel with periodic temperature and dissipative heat. In addition, Chauchan and Kumar [3] studied the steady flow and heat transfer in a composite vertical channel. In [4], Palani and Abbas investigated the combined effects of magneto-hydrodynamics and radiation effect on free convection flow past an impulsively started isothermal vertical plate using the Rosseland approximation. Hussain et al. [5] presented analytical study of oscillatory second grade fluid flow in the presence of a transverse magnetic field and many more.

In all the studies above, the channel walls are assumed to be impervious. This assumption is not valid in studying flows such as blood flow in miniature level where digested food particles are diffused into the bloodstream through the wall of the blood capillary. Hence, due to several other important suction/injection controlled applications, there have been several studies on the convective heat transfer through porous channel; for instance, Umavathi et al. [6] investigated the unsteady flow of viscous fluid through a horizontal composite channel whose half width is filled with porous medium. Ajibade and Jha [7] presented the effects of suction and injection on hydrodynamics

* Corresponding author.
E-mail address: adesanyas@run.edu.ng (S.O. Adesanya).

Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2016.09.016
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of oscillatory fluid through parallel plates. The same authors extended the problem to heat generating/absorbing fluids in [8] while in [9] the effect of viscous dissipation of the free convective flow with time dependent boundary condition was investigated. More recently, Adesanya and Makinde [10] investigated the effect of radiative heat transfer on the pulsatile couple stress fluid flow with time dependent boundary condition on the heated plate. It is well known that the no-slip condition is not realistic in some flows involving Nano-channel, micro-channel and flows over coated plates with hydrophobic substances. In view of this, Adesanya and Gbadeyan [11] studied the flow and heat transfer of steady non-Newtonian fluid flow noting the fluid slip in the porous channel. Other interesting cases on hydromagnetic oscillatory fluid flow under different geometries can be found in [12–20] and references therein.

After careful survey of the literature, it is observed that the effect on suction/injection on the slip flow of oscillatory hydromagnetic fluid through a channel filled with saturated porous medium has not been investigated. Therefore, the specific objective of this paper was to extend the work done in [2], to include the effect of suction/injection at the cold plate. The rest of the paper is organized as follows: Section 2 provides adequate information on the formulation and non-dimensionalization of the problem; in Section 3, the method of solution problem is presented; Section 4 presents the results and discussions while Section 5 concludes the work.

2. Mathematical analysis

Consider the unsteady laminar flow of an incompressible viscous electrically conducting fluid through a channel with slip at the cold plate. An external magnetic field is placed across the normal to the channel. It is assumed that the fluid has small electrical conductivity and the electro-magnetic force produced is also very small. The flow is subjected to suction at the cold wall and injection at the heated wall. We choose a Cartesian coordinate system (x', y') where x' lies along the centre of the channel, and y' is the distance measured in the normal section such that $y' = a$ is the channel's half width as shown in Fig. 1 below.

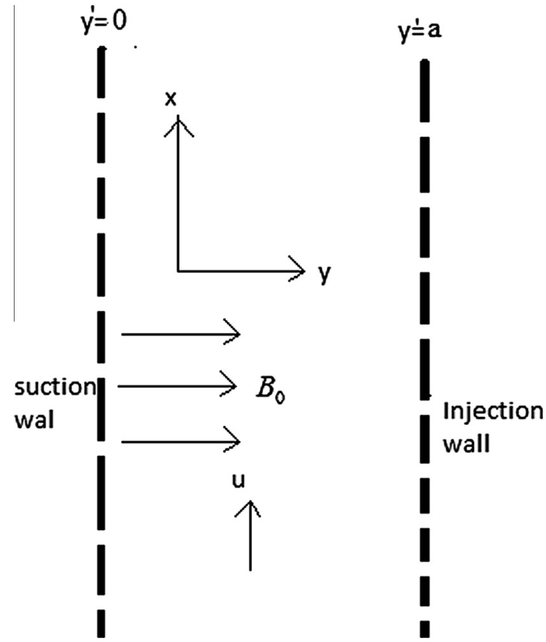

Figure 1 Geometry of the problem.

Under the usual Bousinesq approximation the equations governing the flow are as follows:

$$\frac{\partial u'}{\partial t'} - v_0\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{dP'}{dx'} + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\nu}{K}u' - \frac{\sigma_e B_0^2}{\rho}u' + g\beta(T' - T_0), \tag{1}$$

$$\frac{\partial T'}{\partial t'} - v_0\frac{\partial T'}{\partial y'} = \frac{k_f}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{4\alpha^2}{\rho C_p}(T - T_0) \tag{2}$$

with the boundary conditions $T = T_1$

$$u' = \frac{\sqrt{K}}{\alpha_s}\frac{du'}{dy'},\quad T = T_0 \quad \text{on } y' = 0, \tag{3}$$

$$u' = 0,\ T' = T_1 \quad \text{on } y' = a. \tag{4}$$

where t' – time, u' – axial velocity, v_0 – constant horizontal velocity, ρ – fluid density, P' – fluid pressure, ν – kinematic viscosity, K – porous permeability, σ_e – electrical conductivity, B_0 – magnetic field intensity, g –gravitational acceleration, β – volumetric expansion, C_p – is the specific heat at constant pressure, α – is the term due to thermal radiation, k represents the thermal conductivity, T' fluid temperature and T_0 referenced fluid temperature.

Introducing the dimensionless parameters and variables given in (5)

$$(x, y) = \frac{(x', y')}{h},\ u = \frac{hu'}{\nu},\ t = \frac{\nu t'}{h^2},\ p = \frac{h^2 p'}{\rho\nu^2},$$
$$Gr = \frac{g\beta(T_1 - T_0)h^3}{\nu^2},\ Pr = \frac{\rho C_p \nu}{k},$$

$$\theta = \frac{T - T_0}{T_1 - T_0},\ \delta = \frac{4\alpha^2 h^2}{\rho C_p \nu},\ \gamma = \frac{\sqrt{K}}{\alpha_s h},\ Ha^2 = \frac{\sigma_e B_0^2 h^2}{\rho\nu},$$
$$Da = \frac{K}{h^2},\ s = \frac{v_0 h}{\nu} \tag{5}$$

we obtain the dimensionless Eqs. (6) and (7):

$$\frac{\partial u}{\partial t} - s\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{\partial^2 u}{\partial y^2} - \left(Ha^2 + \frac{1}{Da}\right)u + Gr\theta \tag{6}$$

$$\frac{\partial\theta}{\partial t} - s\frac{\partial\theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2} + \delta\theta \tag{7}$$

with the appropriate boundary conditions (8) and (9)

$$u = \gamma\frac{du}{dy},\quad \theta = 0 \text{ on } y = 0 \tag{8}$$

$$u = 0,\quad \theta = 1 \text{ on } y = 1 \tag{9}$$

In Eqs. (6)–(9), Da is the Darcy parameter, s is the Suction/injection parameter, Ha^2 is Hartmann's number, Gr is the Grashof number, Pr is the Prandtl number, δ is the thermal radiation parameter and γ is the Navier slip parameter.

Please cite this article in press as: J.A. Falade et al., MHD oscillatory flow through a porous channel saturated with porous medium, Alexandria Eng. J. (2016), http://dx.doi.org/10.1016/j.aej.2016.09.016