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ORIGINAL ARTICLE

Boundary layer flow of a Walter's B fluid due to a stretching cylinder with temperature dependent viscosity

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KEYWORDS

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Abstract The present investigation consists of an analytical treatment of a steady boundary layer flow of a Walter's B fluid due to a stretching cylinder with temperature dependent variable viscosity. The heat transfer analysis is also considered. With the help of usual similarity transformations the governing equations have been transformed into nonlinear ordinary differential equations and are solved by a powerful technique homotopy analysis method. Two models of variable viscosity, namely, Reynolds and Vogel's models are taken into account. The convergence is checked by plotting *h*-curves. The emerging parameters are discussed through graphs.

1. Introduction

The ratio of shear stress to the shear strain is known as viscosity. As far as literature survey is concerned a large number of investigations consist of works in which fluid viscosity is considered to be constant. In certain situations, the fluid viscosity does not remain constant. It may vary with distance, temperature or pressure. For example in coal slurries the viscosity of the fluid changes with temperature. In several thermal transport processes, the temperature distribution within the flow field does not remain uniform, i.e., the fluid viscosity may be changed noticeably if large temperature differences exist in the system. Therefore, it is highly desirable to take into account variable viscosity. Fluids that do not obey Newton's law of viscosity are called non-Newtonian fluids. Examples of non-Newtonian fluids are tomato sauce, mustard, mayonnaise, toothpaste, asphalt, lava and ice, mud slides, snow avalanches, etc. Massoudi and Christie [1] have investigated the effects of variable viscosity and viscous dissipation on the flow of a third grade fluid in a uniform pipe. They studied the numerical solutions with the help of straightforward finite difference method. They also discussed that the flow of a fluid-solid mixture is very complicated and may depend on several variables such as physical properties of each phase, size and shape of solid particles. The influence of constant and space dependent viscosity on the flow of a third grade fluid in a pipe has been studied analytically by Hayat et al. [2]. Later on, the approximate and analytical solution of non-Newtonian fluid with variable viscosity has been analyzed by Yursoy and Pakdemirili [3] and Pakdemirili and Yilbas [4]. The pipe flow of non-Newtonian fluid with variable viscosity keeping no slip

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Table 1 Nusselt number for *Re* against *Pr*.

Re/*Pr*	0.1	0.2	0.3	0.4	0.5
0.1	1.24561	1.25054	1.85790	1.85803	1.85814
0.2	1.25071	1.26060	1.86839	1.86865	1.86887
0.3	1.25581	1.27067	1.87888	1.87927	1.87961
0.4	1.26092	1.28076	1.88938	1.88989	1.89035

Table 2 Nusselt number for *A* against *Pr*.

A/*Pr*	0.1	0.2	0.3	0.4	0.5
0.1	1.22637	1.22732	1.2282	1.22901	1.22976
0.2	1.27446	1.27637	1.27812	1.27974	1.28123
0.3	1.32287	1.32574	1.32836	1.33079	1.33303
0.4	1.37157	1.37539	1.37889	1.38212	1.38510

Table 3 Nusselt number for *A* against *Re*.

A/*Re*	0.1	0.2	0.3	0.4	0.5
0.1	1.2345	1.28843	1.34062	1.39122	1.44035
0.2	1.23523	1.28997	1.34303	1.39455	1.44464
0.3	1.23591	1.29140	1.34526	1.39762	1.44859
0.4	1.23654	1.29272	1.34732	1.40046	1.45225

Table 4 Skin friction for *A* against *Re*.

A/*Re*	0.1	0.2	0.3	0.4	0.5
0.1	−6.87921	−6.55554	−6.26807	−6.00921	−5.77344
0.2	−3.74707	−3.56905	−3.41075	−3.2682	−3.13845
0.3	−2.69331	−2.56441	−2.44969	−2.34641	−2.25246
0.4	−2.1597	−2.05575	−1.96321	−1.87991	−1.80419

and partial slip has been investigated analytically by Nadeem and Ali [5] and Nadeem et al. [6]. Recently, Nadeem and Akbar [7] studied the effects of temperature dependent viscosity on peristaltic flow of a Jeffrey-six constant fluid in a uniform vertical tube. Keeping this in mind, we are taking into account temperature dependent viscosity in our study. Stretching is another area of active research. A Newtonian fluid flow over a linear stretching surface was first time considered by Crane [8]. Various aspects of the flow for stretching surfaces have been focused in many investigations [9–17]. Wang [18] studied the steady flow of a viscous and incompressible fluid outside of a stretching hollow cylinder in an ambient fluid at rest. Motivation from abovementioned investigations leads us to consider a steady boundary layer flow of a Walter's B fluid due to a stretching cylinder with temperature dependent variable viscosity. The highly nonlinear problem is transformed into ordinary differential equations with the help of similarity transformations. Renolds and Vogel's models of temperature dependent variable viscosity are considered. The analytical solution is attained using powerful technique homotopy analysis method [6,19–26]. The physical behavior of various parameters is depicted through graphs (see Tables 1–4).

1.1. Description of the problem

Consider steady flow of an incompressible Walter's B fluid flow caused by a stretching tube of radius "a" in the axial direction, where z is the axis along the tube length and r is the axis in the radial direction. The surface of the tube is at temperature T_w and the ambient fluid temperature is T_1, where $T_w > T_1$. The governing equations are

$$\frac{\partial(rw)}{\partial z} + \frac{\partial(ru)}{\partial r} = 0, \tag{1}$$

$$\begin{aligned}\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = {} & \frac{2\eta_0}{r}\frac{\partial u}{\partial r} - \frac{2k_0}{r}u\frac{\partial^2 u}{\partial r^2} - \frac{2k_0}{r}w\frac{\partial^2 u}{\partial z\partial r} \\ & + \frac{\partial}{\partial r}\begin{pmatrix} 2\eta_0\frac{\partial u}{\partial r} - 2k_0 u\frac{\partial^2 u}{\partial r^2} \\ -2k_0 w\frac{\partial^2 u}{\partial z\partial r}\end{pmatrix} - \frac{\partial}{\partial z}\begin{pmatrix}\eta_0\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \\ -k_0 u\left(\frac{\partial^2 u}{\partial r\partial z} + \frac{\partial^2 w}{\partial r^2}\right) \\ -k_0 w\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial z\partial r}\right)\end{pmatrix} \\ & - 2\eta_0\frac{u}{r^2} - 2k_0\frac{u^2}{r^3} - \frac{2k_0}{r^2}\frac{\partial u}{\partial z},\end{aligned} \tag{2}$$

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