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ORIGINAL ARTICLE

Numerical treatments for solving nonlinear mixed integral equation

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Abstract We consider a mixed type of nonlinear integral equation (MNLIE) of the second kind in the space $C[0,T] \times L_2(\Omega), T < 1$. The Volterra integral terms (VITs) are considered in time with continuous kernels, while the Fredholm integral term (FIT) is considered in position with singular general kernel. Using the quadratic method and separation of variables method, we obtain a nonlinear system of Fredholm integral equations (NLSFIEs) with singular kernel. A Toeplitz matrix method, in each case, is then used to obtain a nonlinear algebraic system. Numerical results are calculated when the kernels take a logarithmic form or Carleman function. Moreover, the error estimates, in each case, are then computed.

1. Introduction

Several numerical methods for approximation of the solution of integral equations are known. For FIE, some different methods for continuous kernels were introduced in [1], while for discontinuous kernels, Toeplitz matrix method was presented in [2] and a collocation method was introduced in Brunner [3]. In [4] Yalcinbas applied a Taylor polynomial solution to VFIE. In [5], Diogo and Lima discussed the convergence of spline collocation methods of IE. El-Borai et al. studied the existence and uniqueness solution of NLIE with continuous kernel in [6]. Maleknejad and Sohrabi, in [7], used Legendre polynomials to solve MNLIE. In [8], Matar discussed the fractional semi linear mixed integro-differential equation with continuous kernel in position. Razlighi and Soltanalizadeh used product integration method for solving system of VIE in [9]. Zhao et al. in [10] used collocation methods for fractional integro-differential equations with weakly singular kernel. Many different cases for the IE in linear and nonlinear, with singular kernel are discussed in Abdou et al. [11–13].

Consider the NLMIE in the form

$$\mu\gamma(\phi(x,t)) - \lambda\int_0^t\int_\Omega F(t,\tau)k(|g(x)-g(y)|)\phi(y,\tau)dyd\tau - \lambda\int_0^t V(t,\tau)\phi(x,\tau)d\tau = f(x,t). \quad (1.1)$$

Here, $F(t,\tau)$ and $V(t,\tau)$ are two kernels of continuous functions in time, while $k(|g(x)-g(y)|)$ is discontinuous kernel in position considered to be singular. The constant μ defines the kind of the integral equation, while λ is a numerical parameter that has a physical meaning. $f(x,t)$ is the free term. Ω is the

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domain of the integration and $\gamma(\phi(x,t))$ is the nonlinear function of the unknown function $\phi(x,t)$ in $L_2(\Omega)\times C[0,T], T<1$.

In order to guarantee the existence of a unique solution of Eq. (1.1), we assume the following conditions:

(i) The kernel $k(|g(x)-g(y)|)$ satisfies

$$\left[\int\int_\Omega k^2(|g(x)-g(y)|)dxdy\right]^{\frac{1}{2}} = c, \quad c \text{ is a constant}.$$

(ii) The kernels of VI terms $F(t,\tau)$ and $V(t,\tau)$ satisfy

$$|F(t,\tau)| \leqslant M_1 \quad \forall t,\tau\in[0,T],$$

$$|V(t,\tau)| \leqslant M_2, \quad (M_1 \text{ and } M_2 \text{ are constants}).$$

(iii) The free term $f(x,t)$ and its partial derivatives are continuous in the space $L_2(\Omega)\times C[0,T]$ where

$$\|f(x,t)\| = \max_t\left[\int_\Omega |f^2(x,t)|dx\right]^{\frac{1}{2}} = L, L \quad \text{is a constant}.$$

(iv) The known function $\gamma(\phi(x,t))$ satisfies

$$|\gamma(\phi_1(x,t)) - \gamma(\phi_2(x,t)) \leqslant \epsilon|\phi_1(x,t)) - \phi_2(x,t)|.$$

while the unknown function $\phi(x,t)$ satisfies Lipchitz condition for the first argument and Holder condition for the second argument.

This paper is divided into 5 sections. Section 2, contains two subsections to obtain an NLSFIE from (1.1) using two known methods quadrature and separation methods. Section 3, Toeplitz matrix method is used to get a NLAS. In Section 4, numerical results and estimated errors are computed using previous methods in different examples. In Section 5, general conclusions are deduced.

2. System of Fredholm integral equations

2.1. Quadratic numerical method, see [1]

For representing Eq. (1.1) as a NLSFIE, we divide the time interval [0,T] in the form

$$0 = t_0 < t_1 < t_2 < \ldots < t_l = T.$$

Let $t = t_i$ in Eq. (1.1), we get

$$\mu\gamma(\phi(x,t_i)) - \lambda\int_0^{t_{i1}}\int_\Omega F(t_i,\tau)k(|g(x)-g(y)|)\phi(y,\tau)dyd\tau$$
$$-\lambda\int_0^{t_{i2}} V(t_i,\tau)\phi(x,\tau)d\tau = f(x,t_i). \tag{2.2}$$

Applying quadrature rule, the formula (2.2) reduces to NLSFIE

$$\mu\gamma(\phi_i(x)) - \lambda\sum_{j=0}^{i1}\int_\Omega F_{i,j}k(|g(x)-g(y)|)\phi_j(y)dy$$
$$-\lambda\sum_{j=0}^{i2} V_{i,j}\phi_j(x) = f_i(x). \tag{2.3}$$

Then, we obtain

$$\mu\gamma(\phi_i(x)) - \lambda\frac{h_1}{2}F_{i_1,i}\int_\Omega k(|g(x)-g(y)|)\phi_i(y)dy - \lambda\frac{h_2}{2}V_{i_2,i}\phi_i(x)$$
$$= f_i(x) + \lambda\sum_{j=0}^{i_1-1} w_jF_{i_1,j}\int_\Omega k(|g(x)-g(y)|)\phi_j(y)dy$$
$$+\lambda\sum_{j=0}^{i_2-1} u_jV_{i_2,j}\phi_j(x). \tag{2.4}$$

which can be written as

$$\mu\gamma(\phi_i(x)) - \lambda_i\int_\Omega k(|g(x)-g(y)|)\phi_i(y)dy - \lambda\frac{h_2}{2}V_{i_2,i}\phi_i(x)$$
$$= H_i(x) + E_{l,i}, \quad E_{l,i} = \max_{i_1,i_2} E_{l,i_1}, E_{l,i_2} \tag{2.5}$$

where

$$H_i(x) = f_i(x) + \lambda\sum_{j=0}^{i_1-1} w_jF_{i_1,j}\int_\Omega k(|g(x)-g(y)|)\phi_j(y)dy$$
$$+\lambda\sum_{j=0}^{i_2-1} u_jV_{i_2,j}\phi_j(x) \tag{2.6}$$

and

$$\lambda_i = \frac{\lambda h_1}{2}F_{i_1,i}. \tag{2.7}$$

Here, we used the following notations:

$$\phi_i(x) = \phi(x,t_i), \quad F_{i,j} = F(t_i,t_j), \quad V_{i,j} = V(t_i,t_j),$$
$$f_i(x) = f(x,t_i), i = 0,1,\ldots,l, \quad 0\leqslant j\leqslant i.$$

2.2. Separation of variables

Here, directly we consider the method of separation of variables that has large applications in mechanics of continuous media, see [14], For this, let

$$\phi(x,t) = \phi_n(x)t^n, \quad f(x,t) = f_nt^n, \quad t\in[0,T],$$
$$T<1, \quad n = 1,2,\ldots,N. \tag{2.8}$$

In general, the above assumption may be failed in other science but in continuum mechanics is good to solve the problem directly. The nonlinear term in this case, takes

$$\gamma(\phi(x,t)) = \phi^p(x,t), \quad p = 2orp = \frac{1}{2}. \tag{2.9}$$

Hence, Eq. (1.1) becomes

$$\mu\phi_n^p - \lambda\frac{t^{2m+n+1}}{m+n+1}\int_\Omega k(|g(x)-g(y)|)\phi_n(y)dy$$
$$-\lambda t^{m+n}\beta(m+1,n+1)\phi_n(x) = f_n(x)t^n. \tag{2.10}$$

where, we assume

$$F(t,\tau) = t^m\tau^m, \quad V(t,\tau) = (t-\tau)^m. \tag{2.11}$$

This is a system with NLFIE that can be solved by any suitable method.

3. Toeplitz matrix method, see [2]

In this section, we adapt and apply the TMM to obtain the numerical solution of Eq. (2.5). For this, taking $\Omega = [-a,a]$, then the FIT becomes

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