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ORIGINAL ARTICLE

Comparison between the flow of two non-Newtonian fluids over an upper horizontal surface of paraboloid of revolution: Boundary layer analysis

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Abstract Boundary layer flow of two non-Newtonian fluids past an upper horizontal surface of a paraboloid of revolution (*uhspr*) in the presence of nonlinear thermal radiation is investigated. A new concept of parameterization is introduced to achieve comparison between the flows of both fluids (i.e. switch momentum governing equation from Williamson fluid to Casson fluid). In this study, it is assumed that buoyancy and stretching at the wall induce Casson and Williamson fluid flow over this kind of surface which is neither a perfect horizontal/vertical nor inclined/cone. Influence of space dependent internal heat source is accommodated in the energy equation. The case of unequal diffusion coefficients of reactants A and B (high concentration of catalyst on *uhspr*) is considered. Since chemical reactant B is of higher concentration at the surface more than the concept described as cubic autocatalytic, the suitable schemes are herein described as isothermal quartic autocatalytic reaction and first order reaction. A suitable similarity transformation is applied to reduce the governing equations to coupled ordinary differential equations. These equations along with the boundary conditions are solved numerically by using Runge-Kutta technique along with shooting method. Comparisons of the effects of some parameters on the flow profiles are illustrated graphically and discussed.

1. Introduction

The study of boundary layer flow has attracted the interest of many researchers due to its wide range of important applications in engineering, industries, space science and aviation. Published reports on boundary layer formed on horizontal stretchable surface, impulsively started vertical porous surface, spherical gas bubble, curved surface, circular cylinder, stretching/shrinking sheet, finite flat plate, sliding plate, horizontal melting surface, wing of aircraft, flow of a nanofluid past a permeable stretching/shrinking sheet in the presence of suction/injection, inclined stationary/moving flat plate and over a wedge have been reported in Refs. [1–11]. Considering the influence of space and temperature dependent heat source on the flow of nanofluid within boundary layer on nonlinearly

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permeable stretching sheet over a surface with uniform thickness, Raju et al. [12] reported that an increase in the value of stretching ratio parameter enhances the heat transfer rate and depreciates the friction factor. Historically, the first report on the motion of viscous liquid past a surface called paraboloid was presented by Mather [13] in the year 1961. Thereafter, Lee [14] focused on a thin needle (i.e. object with non-uniform thickness) and presented boundary layer equation governing the motion of an incompressible fluid flow over a thin needle. Cebeci et al. [15] extended the concept to the case of boundary layer flow on slender paraboloid. Miller [16,17] investigated the boundary layer on a paraboloid of revolution and downstream solution for steady viscous flow past a paraboloid. Veldman [18] presented a numerical method for solving Navier-Stokes equations for flow past a paraboloid of revolution. However, stretching of either fluid layers at the free stream or on the wall and viscous term may not be sufficient enough to induce fluid flow over this kind of surface called *uhspr*. Recently, Animasaun [19] deliberated on free convective flow past an upper horizontal surface of a paraboloid of revolution in which gravity is sufficiently strong enough to make the specific weight appreciably different between any two layers of a fluid. In real life, there are many occurrences of fluid (i.e. air, water, nanofluid, Casson fluid and Williamson fluid) flow over the pointed edge of a rocket and aircraft. This part/region of an object is neither a vertical/horizontal surface nor inclined/wedge. To a reasonable extent, an upper half face of an object with variable thickness can be described as a paraboloid of revolution. It is worth mentioning that upper horizontal surface of a paraboloid of revolution resembles the pointed edge of a rocket and aircraft.

In fluid dynamics, fluids may be divided into Newtonian and non-Newtonian. It is a well-known fact that viscous stresses arising from Newtonian fluid flow at every point are directly proportional to the local strain rate. This is not true for non-Newtonian fluid (i.e. the relationship between the viscous stresses arising from the flow is not linear and may even be time-dependent). Three broad classifications of non-Newtonian fluids are time-dependent, time independent and viscoelastic fluids. For the case of time-dependent non-Newtonian fluids, the viscosity is dependent on temperature, shear rate and time. This can be characterized as either thixotropic (time thinning - viscosity decreases with time e.g. paint and yogurt) or rheotactic (time thickening - viscosity increases with time e.g. gypsum paste). Whereas, for the case of time-independent non-Newtonian fluids, the viscosity is only dependent on shear rate and temperature. This can be characterized as shear thinning (pseudoplastic - viscosity decreases with increase in shear rate e.g. shampoo, ketchup, and slurries), shear thickening (dilatant - viscosity increases with an increase in shear rate) and plastic (certain magnitude of shear stress must be applied before flow occurs). Time independent non-Newtonian fluids are those fluids in which the shear rate at a given point is a function of stress at that point only. Animasaun [20] stated that examples are Casson, Bingham, Dilatant and Pseudo-plastic fluid. Among these, non-Newtonian Casson fluid flow is distinct due to the fact that such flow strongly depends on yield stress. An appropriate rheological model was introduced originally by Casson [21] in his research on flow equation for pigment oil suspensions of printing ink. Krishnamurthy et al. [22] explained that certain non-Newtonian fluids are grouped as visco-inelastic fluids, viscoelastic fluids, polar fluids, anisotropic fluids and fluids with micro-structures. Williamson [23] discussed the flow of pseudoplastic materials and proposed a model equation to describe the flow of pseudoplastic fluids and experimentally verified the results. Rheologically, Williamson fluid is a typical example of visco-inelastic fluids. Nadeem et al. [24] stated that Williamson fluid is classified as a non-Newtonian fluid with shear thinning property (i.e., viscosity decreases with increasing rate of shear stress). Ever since suitable models for Casson and Williamson fluid flow have been formulated, experts have reported both flows on various surfaces. Few among related published works are Refs. [25–34].

Scientifically, temperature can be defined as a measure of warmth or coldness of an object/substance while heat can be referred to as a quantity of energy transferred between two bodies of different magnitude of temperatures. The tendency of matter to change in shape, area and volume due to a significant change in temperature through heat transfer is referred to as thermal expansion. Heat can be further described as a source of energy which spontaneously passes between a system and its surroundings. Conduction, radiation and convective (convection) are three basic modes of heat transfer. Convection is heat transfer by way of an intermediate fluid body. Free/natural convection is a mode of heat transfer in which the fluid motion is not generated by any external source but only by density differences in the fluid occurring due to temperature gradients. In free convection, fluid surrounding a heat source receives heat, becomes less dense and rises. The surrounding cooler fluid then moves to replace it. When the cooler fluid is also heated, the process continues, forming convection current; this explains the physics of fluid flow past vertical direction and heat transfer processes. Considering this theory, Shehzad et al. [35] considered flow generated by a bidirectional stretching surface then investigated the case of heat and mass convective at the surface. Motsa and Animasaun [36] reported heat transfer in unsteady mixed convection nanofluid containing gyrotactic microorganisms and nanoparticles using Paired Quasi-Linearization technique. In the research conducted by Abbasi et al. [37] on mixed convection flow of Maxwell nanofluid, it is observed local Nusselt number which is proportional to heat transfer rate reduced for larger heat generation parameter. In another related study, Sandeep et al. [38] investigated 3-dimensional Casson fluid flow over a surface at absolute zero and reported that maximum increase in temperature is observed at an initial unsteady stage where the convective acceleration term in the energy equation plays a minor role. Heat transfer analysis in three-dimensional flow of Maxwell fluid and three-dimensional Oldroyd-B fluid flow in the presence of Cattaneo-Christov heat flux, and heat transfer analysis in the motion of nanofluid along an upper horizontal surface of a paraboloid of revolution with variable thermal conductivity and viscosity have been reported in Refs. [39–41].

Hence, the major objective of this paper was to compare the motion of both Casson and Williamson fluids past an upper horizontal surface of a paraboloid of revolution in the presence of nonlinear thermal radiation; also, to adopt a newly proposed suitable model for quartic autocatalytic kind of homogeneous-heterogeneous in order to accurately investigate this case of chemical reaction in the flow; in addition, to consider the case of unequal diffusion coefficients of reactant A (bulk fluid) and reactant B that is of higher concentration of catalyst at the surface in the presence of space dependent