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ORIGINAL ARTICLE

# Peristaltic pumping of an incompressible viscous fluid in a porous medium with wall effects and chemical reactions

**Gurunath Sankad** *, **Mallinath Dhange**

*Research Centre, Department of Mathematics (Affiliated to Visvesvaraya Technological University, Belagavi, India), B.L.D.E.A.'s V.P. Dr. P.G. Halakatti College of Engineering and Technology, Vijayapur 586103, Karnataka, India*

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**Abstract** This article addresses the effects of homogeneous and heterogeneous chemical reactions on the peristaltic pumping of an incompressible viscous fluid through a porous medium with wall properties. The mean effective coefficient of dispersion has been calculated through long wavelength hypothesis and conditions of Taylor's limit. The effects of various penetrating parameters on mean effective dispersion coefficient are observed graphically.


## 1. Introduction

The dispersion of a solute in a solvent flowing in a channel has applications in physiological fluid dynamics, biomedical and chemical engineering. The basic theory on dispersion was first proposed by Taylor [1–3], who investigated the viscous incompressible laminar flow of a fluid in a circular pipe with dispersion of salute matter. Author believes that, the solute disperses with an equivalent average effective dispersion coefficient, and the dispersion depended on the radius of the tube, coefficient of molecular diffusion and average speed of the flow. Aris [4], Padma and Rao [5], Gupta and Gupta [6], Misra and Ghosh [7], Pal [8], and Sobh [9] investigated the dispersion of a solute in viscous fluid under different limitations. Furthermore, [10–17] extended this analysis to non-Newtonian fluids. Moreover, few authors have studied the dispersion of a solute in a porous medium under various circumstances. Flow through permeable medium has several applications in Geo-fluid dynamics, physiological fluid dynamics and Engineering sciences. The study of flow in permeable media is an immensely vital role for understanding the transport process in kidneys, lungs, and gallbladder with stones. Most of the tissues in the body are deformable permeable media. The proper functioning of such things depends on the flow of blood and nutrients.

Peristalsis is the main technique for transporting many physiological fluids. This motion is involved in ovum movement in the female fallopian tube, the urine segment from the kidney to the bladder, transport of spermatozoa in the efferent ducts in males, advancement of bile in the bile funnel, etc. This mechanism is used in some biomedical devices: hose pumps, finger and roller pumps that use it to force blood, slurries, and other fluids. A few experts have examined the peristaltic transport of different liquids under various circumstances [18–22]. In 1973, the effects of wall on Poiseuille

* Corresponding author.
E-mail addresses: math.gurunath@bldeacet.ac.in (G. Sankad), math.mallinath@bldeacet.ac.in (M. Dhange).

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flow with peristalsis have been examined by [23]. After this study, several investigators have studied the wall effects on different fluids in peristalsis [24–29].

Diffusion, peristalsis and porosity are more essential characteristics in bio-medical, natural and chemical processes. The fluids present in the ducts of living being can be classified as Newtonian and non-Newtonian fluids based on their behaviour. To the best of our knowledge, no attempt has yet been reported to discuss the impact of simultaneous homogeneous and heterogeneous chemical responses on peristaltic stream of an incompressible viscous fluid through a porous medium with wall effects. The application to this problem is moment of nutrients in blood vessels which have peristalsis on its walls [30]. Using $\delta$-approximation, conditions of Taylor's limit and dynamic periphery conditions, the analytic expressions for mean effective scattering coefficient in case of chemical reactions have been obtained. Furthermore, mean dispersion coefficient was calculated numerically. The results are explored for different values of penetrating parameters through graphics.

## 2. Two-dimensional viscous Newtonian porous medium flow model

Consider the peristaltic flow of an incompressible viscous fluid through a porous medium in the 2-dimensional compliant wall channel filled with porous material. The peristaltic wave with speed $c$ produces the flow travelling along walls of the channel. The Cartesian coordinates $x$, $y$ with $x$-axis at the centre of the fluid flow and the homogeneous and heterogeneous reaction effects in the flow analysis. Fig. 1 shows the travelling waves.

The travelling sinusoidal wave is given by the following equation:

$$y = \pm\hbar = \pm\left[d + a\,sin\frac{2\pi}{\lambda}(x - ct)\right], \tag{1}$$

where $a$ is the amplitude and $\lambda$ is the wavelength of the peristaltic wave.

The corresponding flow equations of the present issue are as follows:

$$\frac{\partial \mathcal{U}}{\partial x} + \frac{\partial \mathcal{V}}{\partial y} = 0, \tag{2}$$

$$\rho\left[\frac{\partial}{\partial t} + \mathcal{U}\frac{\partial}{\partial x} + \mathcal{V}\frac{\partial}{\partial y}\right]\mathcal{U} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\mathcal{U} - \frac{\mu}{\bar{k}}\mathcal{U}, \tag{3}$$

$$\rho\left[\frac{\partial}{\partial t} + \mathcal{U}\frac{\partial}{\partial x} + \mathcal{V}\frac{\partial}{\partial y}\right]\mathcal{V} = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\mathcal{V} - \frac{\mu}{\bar{k}}\mathcal{V}, \tag{4}$$

where $\mathcal{U}$ - velocity component in the $x$ direction, $\mathcal{V}$ - the velocity components in the $y$ direction, $\rho$ - the density, $p$ - the pressure, and $\mu$ - the viscosity coefficient.

The equation of the bendable wall movement [23] is given as follows:

$$L(\hbar) = p - p_0, \tag{5}$$

where $L$ - the movement of an expanded membrane by the damping forces and is calculated using the following equation:

$$L = -T\frac{\partial^2}{\partial x^2} + m\frac{\partial^2}{\partial t^2} + C\frac{\partial}{\partial t}. \tag{6}$$

Here, $m$ - mass/unit area, $T$ - the tension in the membrane, and $C$ - the viscous damping force coefficient.

After solving Eqs. (2)–(4) under long-wavelength hypothesis, we get

$$\frac{\partial \mathcal{U}}{\partial x} + \frac{\partial \mathcal{V}}{\partial y} = 0, \tag{7}$$

$$-\frac{\partial p}{\partial x} + \mu\frac{\partial^2 \mathcal{U}}{\partial y^2} - \frac{\mu}{\bar{k}}\mathcal{U} = 0, \tag{8}$$

$$-\frac{\partial p}{\partial y} = 0. \tag{9}$$

The related periphery conditions are

$$\mathcal{U} = 0 \text{ at } y = \pm\hbar. \tag{10}$$

It is presumed that $p_0 = 0$ and the channel walls are inextensible; therefore, the horizontal displacement of the wall is zero and only lateral movement takes place.

$$\text{and } \frac{\partial}{\partial x}L(\hbar) = \mu\frac{\partial^2 \mathcal{U}}{\partial y^2} - \frac{\mu}{\bar{k}}\mathcal{U} \text{ at } y = \pm\hbar, \tag{11}$$

$$\text{where } \frac{\partial}{\partial x}L(\hbar) = \frac{\partial p}{\partial x} = -T\frac{\partial^3\hbar}{\partial x^3} + m\frac{\partial^3\hbar}{\partial x\partial t^2} + C\frac{\partial^2\hbar}{\partial x\partial t}. \tag{12}$$

After solving Eqs. (9) and (10) with conditions (11) and (12), we get

$$\mathcal{U}(y) = \frac{1}{\mu m_1^2}\frac{\partial p}{\partial x}\left[\frac{cosh(m_1 y)}{cosh(m_1\hbar)} - 1\right]. \tag{13}$$

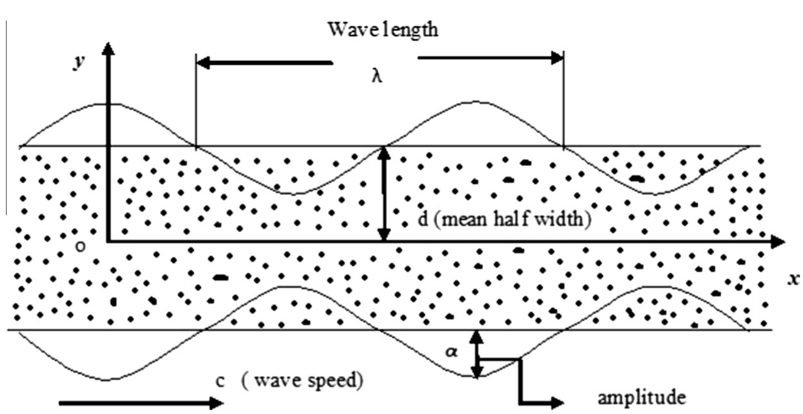


**Figure 1** Geometry of the problem.