ARTICLE IN PRESS

Alexandria Engineering Journal (2016) xxx, xxx–xxx

HOSTED BY ELSEVIER

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

ORIGINAL ARTICLE

Velocity, thermal and concentration slip effects on a magneto-hydrodynamic nanofluid flow

M. Awais [a,*], T. Hayat [b], Aamir Ali [a], S. Irum [a]

[a] Department of Mathematics, COMSATS Institute of Information Technology, Attock 43600, Pakistan
[b] Department of Mathematics, Quaid-I-Azam University, 45320 Islamabad 44000, Pakistan

Received 28 May 2014; revised 19 May 2016; accepted 25 June 2016

KEYWORDS

Slip conditions;
MHD;
Nanofluid;
Streamline analysis

Abstract This article examines the magneto-hydrodynamic (MHD) flow of nanofluid bounded by a stretching surface. The slip conditions are utilized in the present analysis. The involved differential systems are solved for the velocity, temperature and mass fraction. Graphical and numerical results are reported for the analysis of various parameters of interest entering into the modeled problems. Combined effects of thermal and concentration jump are analyzed. Various tables are constructed to show the rheological effects of different physical parameters. Streamlines are plotted showing the rheology for the slip and no slip flow regime. Plots of skin friction are also prepared for the slip and magnetic field effects.

1. Introduction

The study of boundary layer flows has stimulated considerable attention of scientists during the past five decades. This type of flows appears in various engineering applications such as polymer extrusion, continuous casting, glass fiber and paper production, food manufacturing, stretching of plastic films and several other processes. Crane [1] presented earliest study on the steady flow over a stretching sheet. He obtained the closed form solution. Now there is abundant literature available on the flow induced by a stretched surface under various aspects (see few recent studies [2–4]). It is argued that quality of final product in manufacturing processes largely depends upon the rate of cooling. Hence to control cooling system in such process is very important. The simultaneous effects of heat transfer and magnetohydrodynamic (MHD) are useful in order to achieve the final product of desire characteristics. Such considerations are very important especially in the metallurgical processes including the cooling of continuous strips and filaments drawn through a quiescent fluid and purification of molten metals from nonmetallic inclusions. The cooling rate is controlled by drawing the strips in an electrically conducting fluid subject to a magnetic field [5]. A broad description of the MHD/slip/heat transfer effects can be found for instance, to mention the few recent attempts in studies [6–15].

The word "nanofluid" introduced by Choi et al. [16] describes a liquid in which nanometer sized particles are suspended in conventional heat transfer basic fluid. It was noted by Masuda et al. [17] that nanofluids are characterized by enhanced thermal conductivity. No doubt, the conventional fluids used in heat transfer processes such as water, mineral oils and ethylene glycol have low thermal conductivity. Hence several techniques are adopted to improve the thermal

* Corresponding author.
E-mail address: awais_mm@yahoo.com (M. Awais).

Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2016.06.027
1110-0168 © 2016 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V.

Please cite this article in press as: M. Awais et al., Velocity, thermal and concentration slip effects on a magneto-hydrodynamic nanofluid flow, Alexandria Eng. J. (2016), http://dx.doi.org/10.1016/j.aej.2016.06.027

conductivity of such fluids by suspending nano/micro sized particle materials in liquids. Having this in mind, the literature in the mechanics of nanofluids is sizeable now (see [18,19] and many Refs. therein).

The goal of present communication is to venture further in the regime of nanofluids. Thus in this paper we investigate the simultaneous effects of magnetic field and slippage on the stagnation point flow of nanofluid over a stretching surface. The variations of Brownian motion and thermophoresis are particularly analyzed. The no-slip boundary condition (the assumption that a liquid adheres to a solid boundary) is frequently utilized in flow problems of viscous fluids. However, there are cases where such condition is inadequate and slip may occur on the boundary when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions. The fluid flow behavior subject to the slip flow regime greatly differs from the traditional flow. The slip flows under different flow configurations have been studied by many researchers. The solutions are constructed by employing homotopy analysis method ([20–22] and studies mentioned in them). Plots are presented and examined. To the best of our knowledge, this is the first study dealing with slip effects on the flow of a nanofluid.

2. Mathematical analysis

We consider the flow near a stagnation point toward a stretching sheet. The nanofluid is over a stretching sheet. A uniform magnetic field of strength B_0 is applied normally to the stretched surface as shown in Fig. 1.

The temperature and concentration of sheet are denoted by T_w and C_w whereas T_∞ and C_∞ are the ambient values of temperature and concentration respectively. Induced magnetic field effect is not considered in view of small magnetic Reynolds number assumptions. Further, there are slippage effects. The dimensional boundary layer equations can be written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e(x)\frac{du_e(x)}{dx}+\nu\frac{\partial^2 u}{\partial y^2}+\frac{\sigma B_0^2}{\rho_f}(u_e-u) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\frac{(\rho c)_p}{(\rho c)_f}\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

with the following boundary conditions:

$$\begin{aligned}&u=u_w(x)+\gamma_1\frac{\partial u}{\partial y},\quad v=0,\ T=T_w+\gamma_2\frac{\partial T}{\partial y},\\ &C=C_w+\gamma_3\frac{\partial C}{\partial y}\ \text{ at }\ y=0,\\ &u=u_e(x),\quad v=0,\ T\to T_\infty,\ C\to C_\infty\ \text{ as }\ y\to\infty,\end{aligned} \tag{5}$$

in which $u_w(x) = cx$ is the velocity of the stretching surface and $u_e(x) = ax$ is the free stream velocity far away from the stretching surface. Moreover the velocity components (u and v) are along x- and y-axes respectively, ν the kinematic viscosity, T_w the wall temperature, C_w concentration of species near the surface, T_∞ the ambient temperature, C_∞ ambient concentration, σ the electrical conductivity, B_0 the constant magnetic field, p the pressure, D_B the Brownian diffusion coefficient, ρ_f the fluid density, D_T the thermophoretic diffusion coefficient, α_m the thermal diffusivity, $(\rho c)_p$ the effective heat capacity of the nanoparticle material, $(\rho c)_f$ the heat capacity of the fluid and $(\gamma_1, \gamma_2, \gamma_3)$ the slip parameters for velocity, temperature and concentration respectively.

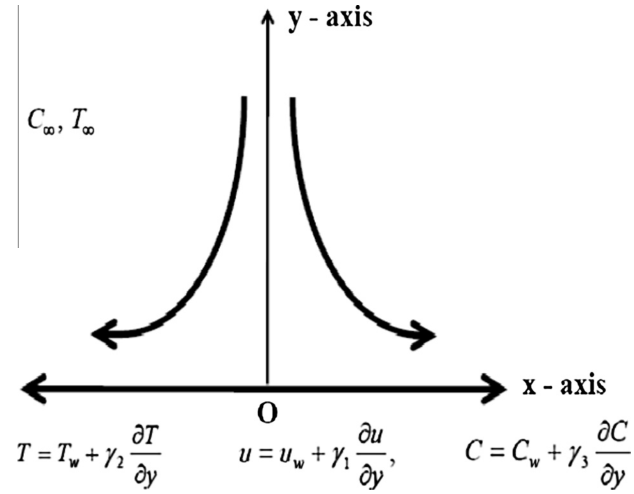

Figure 1 Geometry of the problem.

By introducing

$$\begin{aligned}&\eta=\sqrt{\frac{c}{\nu}}\,y,\quad u=cxf'(\eta),\ v=-\sqrt{c\nu}f(\eta),\\ &\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}\end{aligned} \tag{6}$$

Eq. (1) is satisfied identically and Eqs. (2–5) take the following forms:

$$f'''-(f')^2+ff''+A^2-M^2(f'-A)=0, \tag{7}$$

$$\theta''+\Pr(f\theta'+N_b\phi'\theta'+N_t(\theta')^2)=0, \tag{8}$$

$$\phi''+Lef\phi'+\frac{N_t}{N_b}\theta''=0, \tag{9}$$

$$\begin{aligned}&f(0)=0,\quad f'(0)-S_1f''(0)=1,\\ &\theta(0)-S_2\theta'(0)=1,\ \phi(0)-S_3\phi'(0)=1,\\ &f'(\infty)=A,\ \theta(\infty)=0,\ \phi(\infty)=0.\end{aligned} \tag{10}$$

The quantities $A=\frac{a}{c}$ denote the stretching ratio, M the magnetic parameter, $Le=\frac{\nu}{D_B}$ the Lewis number, $\Pr=\frac{\nu}{\alpha_m}$ the Prandtl number, $N_b=\frac{(\rho c)_p D_B(\phi_w-\phi_\infty)}{(\rho c)_f \nu}$ the Brownian motion parameter, $N_t=\frac{(\rho c)_p D_T(T_w-T_\infty)}{(\rho c)_f T_\infty \nu}$ the thermophoresis parameter, $S_1=\gamma_1\sqrt{c/\nu}$ the velocity slip, $S_2=\gamma_2\sqrt{c/\nu}$ the thermal slip and $S_3=\gamma_3\sqrt{c/\nu}$ the concentration slip.

The Nusselt number Nu and Sherwood number Sh are respectively given by

$$Nu=\frac{xq_w}{k(T_w-T_\infty)},\quad Sh=\frac{xq_m}{D_B(C_w-C_\infty)},$$

in which q_w and q_m denote the surface heat flux and surface mass flux respectively. In dimensionless form

$$Nu/Re_x^{1/2}=-\theta'(0),\ Sh/Re_x^{1/2}=-\phi'(0),$$

where $Re_x = cx/\nu$ denotes the Reynolds number.

Please cite this article in press as: M. Awais et al., Velocity, thermal and concentration slip effects on a magneto-hydrodynamic nanofluid flow, Alexandria Eng. J. (2016), http://dx.doi.org/10.1016/j.aej.2016.06.027