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ORIGINAL ARTICLE

Hydromagnetic boundary layer flow of Williamson fluid in the presence of thermal radiation and Ohmic dissipation

T. Hayat [a,c], Anum Shafiq [a,b,*], A. Alsaedi [c]

[a] *Department of Mathematics, Quaid-i-Azam University 45320, Islamabad 44000, Pakistan*
[b] *Department of Mathematics, Preston University Islamabad Campus, Pakistan*
[c] *Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia*

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Abstract This paper is concerned with the unsteady two-dimensional boundary layer flow of an incompressible Williamson fluid over an unsteady permeable stretching surface with thermal radiation. Effects of electric and magnetic fields are considered. The nonlinear boundary layer partial differential equations are first converted into the system of ordinary differential equations and then solved analytically. Effects of physical parameters such as Weissenberg number, unsteadiness parameter, suction parameter, magnetic parameter, electric parameter, radiation parameter, Prandtl number and Eckert number on the velocity and temperature are graphically analyzed. The expressions of skin friction coefficient and local Nusselt number are presented and examined numerically.

1. Introduction

Magnetohydrodynamic (MHD) is a study of the interaction of electrically conducting fluids and electromagnetic forces. The MHD fluid was first introduced by Swedish Physicist, Alfven [1]. Hartman and Lazarus [2] studied the effects of a transverse uniform magnetic field in the flow of incompressible viscous fluid between two infinite insulating parallel plates. In recent years, the study of magnetohydrodynamic flow of an electrically conducting fluid past a heated surface has attracted the attention of many researchers. This is because of its considerable applications in many engineering problems such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the boundary layer control in aerodynamics and crystal growth. Extensive literature on the MHD flows in presence of applied magnetic field exists now. For example Rashidi et al. [3] considered the MHD flow of nanofluid induced by a rotating disk. Shahzad et al. [4] analyzed the hydromagnetic flow of Maxwell over a bidirectional stretching surface with variable thermal conditions. Turkyilmazoglu [5] analyzed the exact solution of magnetohydrodynamic viscous fluid by a rotating disk. Hayat et al. [6] discussed the buoyancy

* Corresponding author at: Department of Mathematics, Preston University Islamabad Campus, Pakistan.
E-mail address: anumshafiq@ymail.com (A. Shafiq).

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Nomenclature

u, v	axial and transverse velocity components (m/s)	T_w	surface temperature
x	axial coordinate (m)	T_∞	ambient temperature
y	transverse coordinate (m)	a, c	rate constants
ρ	density (kg m^{-3})	ψ	stream function
σ	electrical conductivity (m/s)	θ	dimensionless temperature
K	thermal conductivity (W K^{-1} m^{-1})	f	dimensionless velocity
T	temperature (K)	We	Weissenberg number
μ_0	viscosity (kg m^{-2} s^{-1})	M	magnetic parameter
υ	kinematic viscosity (m^2 s^{-1})	Ec	Eckert number
Γ	time constant	Re	Reynolds number
q_r	radiative heat flux	Pr	Prandtl number
c_p	specific heat (m^2 s^{-2})	E_1	local electric parameter
σ^*	Stefan-Boltzmann constant	A	dimensionless suction/injection parameter
$\mathbf{B}_0$	magnetic field	S	unsteadiness parameter
$\mathbf{J}$	joule current	R_d	radiation parameter
$\mathbf{E}_0$	electric field	τ_w	wall shear stress
k_1	mean absorption coefficient	q_w	rate of heat flux
U_w	stretching velocity	C_f	skin friction coefficient
V_w	suction/injection parameter	Nu_x	Nusselt number
T_0	reference temperature	$C_i(i = 1 - 5)$	constants

driven MHD flow of thixotropic fluid. They also examined the effects of thermophoresis and Joule heating in this investigation. An applied magnetic field effect in natural convection flow of nanofluid is studied by Sheikholeslami et al. [7]. Dandapat and Mukhopadhyay [8] discussed the stability characteristics of a thin conducting liquid film flowing and a non-conducting plane in the presence of electromagnetic field. Hayat et al. [9] investigated the series solution of magnetohydrodynamic axisymmetric flow of third grade fluid between porous disks with heat transfer. Unsteady magnetohydrodynamic mixed convection stagnation point flow of viscoelastic fluid towards a vertical surface is discussed by Ahmad and Nazar [10]. Magnetohydrodynamic Jeffery–Hamel nanofluid flow through non-parallel walls is investigated analytically by Hatami et al. [11]. They used different base fluids and nanoparticle. Sheikholeslami et al. [12] considered the effect of MHD in an inclined L-shape enclosure filled with nanofluid. Entropy analysis for a MHD nanofluid flow over a stretched surface is analyzed by Abolbashari et al. [13]. Rashidi et al. [14] made an analytical method for solving MHD convective and slip flow/-due to a rotating disk.

The effect of radiation on flow and heat transfer is important in the framework of space technology and processes involving high temperature. In the presence of a magnetic field, the radiative flows of an electrically conducting fluid with high temperature are encountered in electrical power generation, solar power technology, astrophysical flows, nuclear engineering applications and other industrial areas. The flows of viscous and non-Newtonian fluids through various aspects and thermal radiation have been examined by some researchers. For instance, Rashidi et al. [15] analytically studied the MHD mixed convective heat transfer in flow of incompressible viscoelastic fluid past a permeable wedge with thermal radiation. Bhattacharyya et al. [16] analyzed the effects of thermal radiation in micropolar fluid flow and heat transfer over a porous shrinking surface. Makinde and Ogulu [17] presented a numerical study for the effect of temperature dependent viscosity in free convective flow past a vertical porous plate. Here the influences of magnetic field, thermal radiation and a first order homogeneous chemical reaction are considered. Hayat et al. [18] investigated the effect of Joule heating in the flow of third-grade fluid over a radiative surface. Motsumi and Makinde [19] reported the numerical solutions for the effect of thermal radiation and viscous dissipation in boundary layer flow of nanofluids towards a permeable moving flat plate. Makinde [20] analyzed the hydromagnetic mixed convection heat and mass transfer flow of an incompressible Boussinesq fluid past a vertical porous plate with constant heat flux and thermal radiation. Shahzad et al. [21] studied the heat and mass transfer characteristics in three-dimensional flow of an Oldroyd-B fluid with thermal radiation.

The aim of present study wasand discussion are included to explore the hydromagnetics effects in the boundary layer flow of Williamson fluid over an unsteady permeable stretching sheet. Mathematical analysis has been carried out in presence of thermal radiation and Ohmic dissipation. This paper is structured as follows. The flow problem is formulated in Section two. Sections three and four deal with the series solutions and convergence analysis respectively. The series solutions are developed through homotopy analysis method [22–32]. Results and discussion are included in Section five and Section six consists of conclusions.

2. Mathematical formulation

Here the two-dimensional boundary layer flow of an incompressible electrically conducting Williamson fluid flow over unsteady porous stretching surface is considered. The flow is due to stretching sheet from a slit with the application of two equal and opposite forces in such a manner that the velocity field in flow direction is linear (see Fig. 1). Frictional heating in view of viscous dissipation is taken into account. The

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