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ORIGINAL ARTICLE

Generalized thermoelastic diffusion in a thick circular plate including heat source

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Abstract The present paper is aimed at studying thermoelastic diffusion interactions in a thick circular plate of infinite extent and finite thickness subjected to an axisymmetric heat supply and a heat source in the context of Lord–Shulman theory of generalized thermoelastic diffusion. The upper and the lower surfaces of the thick plate are traction free and the chemical potential is assumed to be a known function of time. Integral transform techniques are used to find the analytic solution in the transform domain. Mathematical model is prepared for Copper material plate and the numerical results are discussed and illustrated graphically.

1. Introduction

The behaviour of elastic bodies under the influence of non-uniform temperature fields is studied in the theory of thermoelasticity. Widespread interest was developed in this theory due to its applications in diverse engineering problems such as problems related to modern aircraft structure, design of nuclear reactors, and ship building processes. The classical theory of dynamic thermoelasticity introduced by Biot [1] takes into account the coupling between temperature and strain fields. However, it involves a paradox that the thermal disturbances propagate with infinite speeds. To remove this paradox generalized thermoelasticity theories have been developed. Lord and Shulman [2] introduced the theory of generalized thermoelasticity with one relaxation time, which is the time needed for the acceleration of thermal wave. Thus, making the heat conduction equation hyperbolic in nature predicts finite wave propagation.

Diffusion is one of several transport phenomena that occurs in nature. The diffusion in solids can be explained in two ways: either according to Fick's laws where diffusion is the passive movement of molecules or particles along a concentration gradient, or from regions of higher to regions of lower concentration or from the atomic point of view where diffusion is considered as a result of random walk of the diffusing particles. Thermoelastic diffusion in an elastic solid takes place due to the coupling between the fields of temperature, mass diffusion and strain. Heat and mass exchange takes place during thermoelastic diffusion in an elastic solid. Nowacki [3–6] developed the theory of thermoelastic diffusion within the context of classical coupled thermoelasticity and studied some dynamical problems of diffusion in solids. The theory of Nowacki uses Fick's law.

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It predicts infinite speed of wave propagation which is a major drawback. Sherief et al. [7] developed the theory of generalized thermoelastic diffusion in an elastic solid in the context of Lord–Shulman theory (TEDLS), which allows finite speeds of propagation of thermal disturbances. In this theory, Fick's law was modified to include the time derivative of the flux of the diffusive mass. Kumar and Kansal [8] introduced the generalized theory of thermoelastic diffusion in the context of Green-Lindsay theory by introducing thermal relaxation time parameters and diffusion relaxation parameters into the governing equations. Olesiak and Pyryev [9] reported influence of cross effects while studying thermoelastic diffusion in an elastic cylinder. They observed that the thermal excitations result in an additional mass concentration and vice-versa. A problem of thick plate in generalized thermoelastic diffusion theory was discussed by Sherief and El-Maghraby [10]. Sherief and Saleh [11] have considered the problem of thermoelastic half-space in the context of generalized thermoelastic diffusion with one relaxation time. Aouadi [12] studied a one-dimensional problem for an infinitely long solid cylinder and El-Maghraby [13,14] solved two-dimensional problems for a thick plate and half-space under the action of body forces. Elhagary [15] has discussed one dimensional problem of generalized thermoelastic diffusion for a long hollow cylinder. Allam [16] studied a stochastic half-space problem in the theory of generalized thermoelastic diffusion including heat source. Tripathi et al. [17] discussed a two dimensional dynamic problem of generalized thermoelasticity in Lord–Shulman theory for a thick circular cylinder with internal heat generation. Tripathi et al. [18,19] studied problems of generalized thermoelastic diffusion in a thick circular plate and a half-space under axisymmetric distributions. Recently, Allam et al. [20] discussed a stochastic thermoelastic diffusion problem in an infinitely long annular cylinder.

In this work, we have extended our work [18] by including heat source in a generalized thermoelastic diffusion problem for a thick circular plate of finite length and infinite extent subjected to an axisymmetric heat supply whose boundaries are traction free. The chemical potential considered is time dependent. The classical coupled thermoelastic diffusion theory (TEDCT) is recovered as a special case. Analytic solutions for temperature, concentration, chemical potential, displacement and stresses are obtained in the Laplace transform domain. Numerical inversion of Laplace transforms are performed using Gaver-Stehfest Algorithm [21–23] which is considerably more stable and computationally efficient than inversion using the discrete Fourier transform [24] and all integrals were evaluated using Romberg's integration technique [25] with variable step size. A mathematical model is prepared for copper material plate and results are discussed along with the graphical representation. The results presented here may be useful in many engineering problems related to diffusion in isotropic elastic solids with internal heat generation.

2. Governing equations

The governing equations for the generalized thermoelastic diffusion in an isotropic medium in the absence of body forces and in the presence of internal heat generation are given as follows [7]:

1. The equation of motion is given by,

$$\rho \ddot{u}_i = \mu u_{i,jj} + (\lambda + \mu) u_{j,ij} - \beta_1 T_{,i} - \beta_2 C_{,i} \tag{1}$$

where T is the absolute temperature, C is the concentration of the diffusive material, ρ is the density, λ and μ are Lame's constants and β_1 and β_2 are material constants given by $\beta_1 = (3\lambda + 2\mu)\alpha_t$ and $\beta_2 = (3\lambda + 2\mu)\alpha_c$, where α_t is the coefficient of linear thermal expansion, and α_c is the coefficient of linear diffusion equation.

2. The energy equation is given by,

$$kT_{,ii} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)(\rho C_E T + T_0 \beta_1 e + T_0 a C) - \rho\left(1 + \tau_0 \frac{\partial}{\partial t}\right) Q \tag{2}$$

where k is the thermal conductivity of the medium, C_E is the specific heat at constant strain, τ_0 is the thermal relaxation time, T_0 is the reference temperature chosen such that $|(T - T_0)/T_0| << 1$, a is the measure of thermoelastic diffusion effect, $e = u_{i,i}$ is the cubical dilatation, where $u_i, i = 1, 2, 3$ are the components of the displacement vector and Q is the amount of heat resulted from the internal heat generation.

3. The equation of mass diffusion is given by,

$$D\beta_2 e_{,ii} + DaT_{,ii} + \left(\frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2}\right) C = DbC_{,ii} \tag{3}$$

where D is the diffusion coefficient, b is a measure of diffusive effect and τ is the diffusion relaxation time.

4. The constitutive equations are,

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}(\lambda e - \beta_1(T - T_0) - \beta_2 C) \tag{4}$$

$$P = -\beta_2 e + bC - a(T - T_0) \tag{5}$$

where σ_{ij}, $i, j = 1, 2, 3$ are the components of stress tensor, P is the chemical potential of the material per unit mass and e_{ij}, $i, j = 1, 2, 3$ are the components of the strain tensor, given by

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad i, j = 1, 2, 3 \tag{6}$$

3. Formulation of the problem

We take the axis of symmetry as the z axis and the origin of the system of co-ordinates is at the middle plane between the upper and lower faces of the plate. The problem is studied using the cylindrical polar co-ordinates (r, φ, z). Due to the rotational symmetry about the z axis, all quantities are independent of the co-ordinate φ.

Consider a thick circular plate of thickness $2b$ occupying the space D defined by

$$D = \{(r, \varphi, z): \ 0 \leqslant r \leqslant \infty, -b \leqslant z \leqslant b\}$$

Let the thick circular plate be subjected to an axisymmetric heat supply dependent on the radial and axial directions of the cylindrical co-ordinate system. For time $t > 0$, heat is generated within the plate at the rate $Q(r, z, t)$. The initial temperature in the thick plate is given by a constant temperature T_0 and the heat flux $g_0 F(r, z)$ is prescribed on the upper and lower boundary surfaces. Under these conditions, the thermoelastic