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ORIGINAL ARTICLE

Thermal analysis of straight rectangular fin using homotopy perturbation method

Pranab Kanti Roy *, **Ashis Mallick**

Department of Mechanical Engineering, Indian School of Mines, Dhanbad 826004, Jharkand, India

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Abstract In this study a simple and accurate semi-analytical method called Homotopy Perturbation Method (HPM) is used for solving nonlinear energy equation in a straight rectangular fin. The thermal conductivity and surface emissivity are considered as temperature dependent with some constant source of internal heat generation. The problem is solved for two main cases of thermo geometric fin parameter $N_c = 1$ and for $N_c = 0.25$. The results are presented for the temperature distribution, efficiency and optimum dimensionless parameters are effective and convenient for practical fins. Also it is found that this method can achieve more suitable results as compared to the other methods available in the literature.

1. Introduction

The fins are extended surface that are used to dissipate heat from the primary surface to the surrounding environment [1]. Heat transfer through the straight is common because of their low manufacturing cost and simplicity. The fins are sometimes subjected to both convective and radiative environments [2]. Under the circumstances of both convective and radiative modes of heat transfer the fin may generate heat internally due to passage of electric current as in electric filament or due to an atomic or chemical reaction as in an atomic reactor [3–6]. The internal heat generation may be assumed to be at constant rate with respect to the volume of fin. But to utilize the fin materials effectively the model includes the thermal conductivity varies linearly with temperature and in most cases the real surface of the fin material varies linearly with temperature. The energy equation for a convective–radiative fin along with heat generation with two variable thermal properties results in highly nonlinear terms. The resulting equation does not admit the exact solution. Consequently the energy equation with nonlinear terms is solved either numerically or using a variety of approximate analytical methods. Many different methods have been introduced to solve nonlinear problems, the Homotopy analysis method (HAM), Galerkin method (GM), Spectral collocation method (SCM), Adomian decomposition method (ADM). Homotopy perturbation method (HPM) is one of the semi analytical methods for solving nonlinear boundary value problem introduced by He [7]. An HPM possesses all the advantages of perturbation method and also it is independent of assumption of small parameter. As compared to the Adomian decomposition method, Homotopy perturbation method does not require the calculation of Adomian polynomials but it requires only the initial approximation [8–10]. Also the method does not require determination of 'h-curves' which

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* Corresponding author. Tel.: +91 9609551679.
E-mail addresses: pranab38_skd@yahoo.com (P.K. Roy), mal123_us@yahoo.com (A. Mallick).

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Nomenclature

N_r	radiation–conduction parameter
C	constant which represents the temperature
k	temperature dependent thermal conductivity (W/(mK))
k_a	thermal conductivity corresponding to ambient condition (W/(mK))
ε_s	the surface emissivity corresponding to radiation sinks temperature, T_s
T	temperature (K)
P	fin perimeter (m)
T_b	fin's base temperature (K)
T_a	convection sink temperature (K)
T_s	sink temperature for radiation (K)
b	length of the fin (m)
x	axial coordinate of the entire fin (m)
A_c	cross-sectional area of the entire fin (m^2)
X	dimensionless axial coordinate
A	thermal conductivity parameters
B	the surface emissivity parameters
Greek symbols	
α	slope of the thermal conductivity-temperature curve (K^{-1})
β	slope of the surface emissivity-temperature curve (K^{-1})
θ	dimensionless temperature of the fin
θ_a	dimensionless convection sinks temperature
θ_s	dimensionless radiation sinks temperature
σ	Stefan–Boltzmann constant (W/(m^4 K^4))
ε	emissivity

makes it different from the Homotopy Analysis method. Bhowmik et al. [11] predicted the dimensions of rectangular and hyperbolic fins with variable thermal properties. Hazarika et al. [12] established an analytical method to predict the performance and design parameters of T-shape fin with simultaneous heat and mass transfer. Several researchers applied Collocation method (CM) to obtain the analytical solution of unsteady motion of fluid particles in conjunction with other numerical technique [13,14]. Rahimi-Gorji et al. [15] used Galerkin Method to obtain the heat transfer characteristics of micro channel heat sink cooled by different nanofluids in porous media. Pourmehran et al. [16] studied the thermal analysis of a fin shaped micro channel heat sink cooled by different nanofluids using least square method. Spectral collocation method has been used to study the performance parameter of simple and complex cross section of moving rod in thermal processing of continuous casting and rolling [17,18]. Sun et al. [19] used the Lagrange interpolation polynomials to approximate the temperature distribution at the spectral collocation points of nonlinear heat transfer problems. Ma et al. [20] presented the spectral collocation method to predict the thermal performance of porous fin with temperature dependent heat transfer coefficient, surface emissivity and internal heat generation.

From the above discussion it is clear that in fin design application the selection of choice of conductive–convective parameter is very important for practicing engineer. The practical fins work at low values of conductive convective fin parameter. HPM is used to study the variation of surface emissivity, thermal conductivity parameter, heat generation number on the temperature distribution and efficiency of fin working under practical range of operating condition.

2. Mathematical formulations

Let us consider rectangular fin geometry of various details as shown in Fig. 1. The thermal conductivity k and surface emissivity ε of the fin materials are temperature dependent. The heat generated inside the fin material is assumed to be at the constant rate q. The fin is assumed to be insulated at the free end side and the effect of heat transfer in vertical direction is neglected. The steady state one dimensional energy equation is given by

$$\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right]-\frac{hP(T-T_a)}{A_c}-\frac{\varepsilon(T)\sigma P(T^4-T_s^4)}{A_c}+q=0 \qquad (1)$$

With the insulated boundary condition

$$\frac{dT}{dX}=0 \quad \text{at} \quad x=0 \qquad (2a)$$

$$T=T_b \quad \text{at} \quad x=b \qquad (2b)$$

For the better utilization of the fin materials the thermal conductivity of fin material is assumed to be linear function of temperature. Also the emissivity of all real surfaces is not constant, and it sometimes varies linearly with temperature. Therefore both of the parameters can written as below

$$k(T)=k_a[1+\alpha(T-T_a)] \qquad (3)$$

$$\varepsilon(T)=\varepsilon_s[1+\beta(T-T_s)] \qquad (4)$$

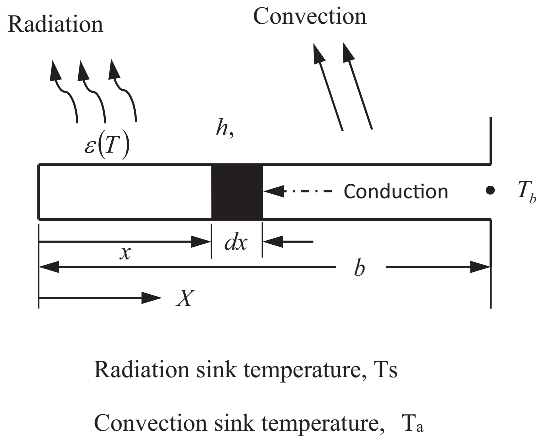

Figure 1 The geometry of straight rectangular fin.