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ORIGINAL ARTICLE

Two parameters Lie group analysis and numerical solution of unsteady free convective flow of non-Newtonian fluid

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Abstract The two-dimensional unsteady laminar free convective heat and mass transfer fluid flow of a non-Newtonian fluid adjacent to a vertical plate has been analyzed numerically. The two parameters Lie group transformation method that transforms the three independent variables into a single variable is used to transform the continuity, the momentum, the energy and the concentration equations into a set of coupled similarity equations. The transformed equations have been solved by the Runge–Kutta–Fehlberg fourth-fifth order numerical method with shooting technique. Numerical calculations were carried out for the various parameters entering into the problem. The dimensionless velocity, temperature and concentration profiles were shown graphically and the skin friction, heat and mass transfer rates were given in tables. It is found that friction factor and heat transfer (mass transfer rate) for methanol are higher (lower) than those of hydrogen and water vapor. Friction factor decreases while heat and mass transfer rate increase as the Prandtl number increases. Friction (heat and mass transfer rate) factor of Newtonian fluid is higher (lower) than the dilatant fluid.

1. Introduction

Studies of non-Newtonian fluids flow have received much attention and importance than Newtonian fluids, due to their various technological and industrial applications. If there is a

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Nomenclature

A	constant
B	constant
$\overline{C}$	concentration
C_f	skin friction factor
D	mass diffusivity
f	dimensionless velocity functions
g	acceleration due to gravity
Gr	Grashof number
K	consistency coefficient
L	reference length
N	buoyancy ratio
n	exponents identifying non-Newtonian behavior
Nb	Brownian motion
Nu	Nusselt number
Pr	Prandtl number
Ra	Rayleigh number
Re	Reynolds number
Ri	Richardson number
Sc	Schmidt number
Sh	Sherwood number
$\overline{T}$	temperature
$\bar{t}$	time
$\bar{u}$	velocity in the $\bar{x}$-direction
U	reference velocity
$\bar{v}$	velocity in the $\bar{y}$-direction
$\bar{x}$	distance along the surface
$\bar{y}$	distance normal to the surface

Greek letters

α	molecular diffusivity
α_i	constant
β_i	constant
β_C	coefficient of mass expansion
β_T	coefficient of thermal expansion
ϕ	dimensionless concentration
η	similarity variable
θ	dimensionless temperature
ρ	density of the ambient fluid
ψ	stream function

Subscript, Superscript

∞	conditions far away from the surface
$'$	differentiation with respect to η
w	wall

nonlinear relationship between stress and the rate of strain, a fluid could be said to be non-Newtonian, such that the viscosity is a function of variables such as temperature and density. Examples of non-Newtonian fluids are mud, paste, personal care products, ice cream, paints, oils, cheese, asphalt. Many biological fluids with higher molecular weight components are also non-Newtonian in nature. Non-Newtonian fluids have many applications in industries such as coal slurries, polymers solution or melts, paints, greases, drilling mud and hydrocarbon oils [1–6]. For describing the properties of non-Newtonian fluids, there are many models. These models or constitutive equations, however cannot describe all the behaviors of these non-Newtonian fluids, for example, normal stress differences, shear thinning or shear thickening, stress relaxation, elastic effects and memory effects. A rigorous study of the boundary layer flow and heat transfer of different non-Newtonian fluids past a stretching sheet was required due to its immense industrial applications. Many authors have embarked on the study of non-Newtonian flow for various reasons and using different methods of solution. Advancement in the study of various non-Newtonian fluids has been made even by many investigators (see Refs. [7–12]). Recently, Hayat et al. [13] studied third grade fluid over an unsteady permeable stretching sheet using HM. Shafiq et al. [14] studied magnetohydrodynamic axisymmetric flow of a third grade fluid between two porous disks. It is to certify that there are several limitations for the applied model for the analysis based on the non-Newtonian fluids; however, these models are still useful tools and employed in different analysis in order to formulate and study the roles of non-Newtonian fluids in different conditions and geometries (See these related papers [15–19]).

The steady viscous incompressible flow of a non-Newtonian power-law fluid on a two-dimensional body in the presence of a magnetic field was studied by Sarpkaya [20] and Djukic [21,22]. Andersson et al. [23] have considered the steady MHD flow of a power-law fluid over a linearly stretching surface. The flow and heat transfer of a power-law fluid over a uniform moving surface with a constant parallel free stream in the presence of a magnetic field have been studied by Kumari and Nath [24]. Liao [25] has obtained an analytical solution of the MHD of a non-Newtonian power-law fluid over a linearly stretching surface. Among the most popular useful models for non-Newtonian fluids is the power-law or Ostwald-de Waele model [26]. The steady flow and heat transfer of a viscous incompressible power-law fluid over a rotating infinite disk have been investigated by Ming et al. [27]. This model is a simple nonlinear equation of state for inelastic fluids which includes linear Newtonian fluids as a special case. The power-law model provides an adequate representation of many non-Newtonian fluids over the most important range of shear rates. This, together with its apparent simplicity, has made it a very attractive model in both analytical research and numerical research. Pascal [28] presented similarity solutions to some unsteady flows of non-Newtonian fluids of power law behavior. Pascal and Pascal [29] studied the nonlinear effects on some unsteady non-Darcian flows through porous media. Unsteady forced convection heat transfer on a flat plate embedded in the fluid-saturated porous medium with inertia effect and thermal dispersion is investigated by Wen and Hsiao [30]. Israel-Cookey et al. [31] discussed the influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction. Recently, Chiem and Zhao [32]