



ORIGINAL ARTICLE

Fractional calculus approach to study temperature distribution within a spinning satellite



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Abstract This paper deals with the temperature distribution within spinning satellites and problem is formulated in terms of fractional differential equation. Applying fractional calculus approach, solution of this equation is obtained in terms of Wright generalized hypergeometric function, a generalization of exponential function.

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1. Introduction

A spinning satellite is a satellite which has the motion of one axis held (relatively) fixed by spinning the satellite around that axis, using the gyroscopic effect. The attitude of a satellite or any rigid body is its orientation in space.

- The entire satellite including its antenna(s) spins.
- This means that the satellite's attitude is very stable because the satellite as a whole is acting as a gyroscope.

- Spinning can be achieved using rods with coils of wire around them.
- Current passing through the rods, creating magnetic fields around the wires.
- When this rod's magnetic field interacts with Earth's magnetic field, the rod begins to spin.
- An advantage of spin stabilized is it requires very little power.
- A disadvantage is that the solar panels cannot all collect power all the time because they will spin, so they don't face the sun once every rotation.
- Also, the instruments can only take measurements in one direction once every rotation.

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Liu et al. [11] studied temperature distribution using improved thermal network model. The conventional thermal network model for satellite surface temperature distribution is directly solved by a new solution method on the basis of

the Monte Carlo Ray Tracing (MCRT) method. The solar direct incidence area, the solar radiation transfer coefficient, the infrared radiation transfer coefficient, and the network conduction and radiation coefficients are calculated by Monte Carlo statistical analysis rather than the Gebhart approach. The advantage of the MCRT method is that the surface material characteristics are taken into account in solving the process. The effect of absorptivity and emissivity for temperature distributions is analyzed in detail. Numerical simulation is carried out for the temperature distribution of the satellite surface with different solar incidences of round angle and zenith angle. In the unsteady stage, the cooling velocity of the main body surface is always smaller than that of the solar array surface and the maximum temperature of the main body is larger than the solar arrays. Under the same absorptivity–emissivity ratio, the solar incidence angle has a great effect on the uniform character of temperature distribution.

The temperature distribution in satellite is also studied by Venkataraman and Egalon [27], and Liu et al. [14].

The fractional Calculus is mathematics which deals with generalization of ordinary integrals and derivatives of integer orders to arbitrary ones. The role of this kind of calculus is to solve problems of complex systems that appear in various fields of sciences [25,9,17]. Recently, Prajapati and Kachhia [22] studied fractional model of temperature distribution and heat flux in the semi infinite solid. Kachhia and Prajapati [8] obtained solution of fractional partial differential equation arises in study of heat transfer through diathermanous materials. In the fractional modeling the fractional derivatives appear naturally in dealing with generalization of the existing classical models [1]. Machado and Galhano [15] used fractional calculus to study inductive phenomena based on the skin effect. Many applications of fractional calculus can be found in other diverse fields, etc. [2,7,5].

In the design of artificial satellites, it is important to determine the temperature distribution on the spacecraft’s surface. An interesting special case is the temperature fluctuation in the skin due to the spinning of the vehicle. Lee et al. [10] studied analysis the unsteady-state temperature distribution of micro-satellite under stabilization effects. Liu et al. in [13] gave improved solution to thermal network problem in heat-transfer analysis of spacecraft, temperature distribution of satellite surfaces affected by solar absorptivity in [14] and numerical simulation on antenna temperature field of complex structure satellite in solar simulator in Liu et al. [12]. Yang et al. [28] studied thermal analysis for folded solar array of spacecraft in orbit.

If the craft is thin-walled so that there is no radial dependence, Hrycak [6] showed that he could approximate the nondimensional temperature field at the equator of the rotating satellite by

$$\frac{d^2 T}{d\eta^2} + b \frac{dT}{d\eta} - c \left(T - \frac{3}{4} \right) = -\frac{\pi c}{4} \frac{F(\eta) + \frac{b}{4}}{1 + \frac{\pi\beta}{4}}, \tag{1.1}$$

where

$$b = \frac{4\pi^2 r^2 f}{a}, \quad c = \frac{16\pi S}{\gamma T_\infty} \left(1 + \frac{\pi\beta}{4} \right), \quad T_\infty = \left(\frac{s}{\pi\sigma\epsilon} \right)^{\frac{1}{4}} \left(\frac{1 + \frac{\pi\beta}{4}}{1 + \beta} \right)^{\frac{1}{4}}, \tag{1.2}$$

$$F(\eta) = \begin{cases} \cos(2\pi\eta), & \text{for } 0 \leq \eta \leq \frac{1}{4}, \\ 0 & \text{for } \frac{1}{4} \leq \eta \leq \frac{3}{4}, \\ \cos(2\pi\eta) & \text{for } \frac{3}{4} \leq \eta \leq 1, \end{cases} \tag{1.3}$$

a is the thermal diffusivity of the shell, f is the rate of spin, r is the radius of spacecraft, S is the net direct solar heating, β is the ratio of the emissivity of the interior shell to the emissivity of the exterior surface, ϵ is the overall emissivity of the exterior surface, γ is the satellite’s skin conductance and σ is the Stefan–Boltzmann constant. The independent variable η is the longitude along the equator with the effect of rotation subtracted out ($2\pi\eta = \phi - 2\pi ft$). The reference temperature T_∞ equals the temperature that the spacecraft would have if it spun with infinite angular speed sp that the solar heating would be uniform around the craft. We nondimensionalized the temperature with respect to T_∞ . We begin by introducing the new variables

$$y = T - \frac{3}{4} - \frac{\pi\beta}{16 + 4\pi\beta}, \quad v_0 = \frac{2\pi^2 r^2 f}{a\rho_0}, \quad A_0 = -\frac{\pi\rho^2}{4 + \pi\beta}$$

and $\rho_0^2 = c$ so that Eq. (1.1) becomes [4]

$$\frac{d^2 y}{d\eta^2} + 2\rho_0 v_0 \frac{dy}{d\eta} - \rho_0^2 y = A_0 F(\eta) \tag{1.4}$$

The following well-known facts are considered for studying the temperature distribution within a spinning satellite.

The Laplace Transform is defined [26] as

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t) dt \quad (Re(s) > 0) \tag{1.5}$$

The Caputo fractional derivative of order α is defined [17] as

$$D_t^\alpha f(t) = \frac{1}{\Gamma(n - \alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t - \tau)^{n-\alpha-1}} d\tau \quad (n - 1 < \alpha < n) \tag{1.6}$$

The Laplace Transform of the Caputo fractional derivative is defined as [17]

$$\int_0^\infty e^{-st} D_t^\alpha f(t) dt = s^\alpha f(s) - \sum_{j=0}^{n-1} s^{\alpha-j-1} f^{(j)}(0) \quad (n - 1 < \alpha < n) \tag{1.7}$$

The convolution product of two functions $f(t)$ and $g(t)$ is defined [3] as

$$f(t) * g(t) = \int_0^t f(t - x)g(x)dx = \int_0^t f(x)g(t - x)dx \tag{1.8}$$

Theorem 1.1. Let $f(t)$ and $g(t)$ be two functions defined for $t \geq 0$ and $\bar{f}(s)$ and $\bar{g}(s)$ be the Laplace transforms of $f(t)$ and $g(t)$ respectively. Then

$$L^{-1}\{\bar{f}(s)\bar{g}(s)\} = f(t) * g(t) \tag{1.9}$$

The Wright generalized hypergeometric function is defined in Mathai et al. [16] as

$${}_p\Psi_q(z) = {}_p\Psi_q \left[z \left| \begin{matrix} (a_p, A_p) \\ (b_q, B_q) \end{matrix} \right. \right] = \sum_{k=0}^\infty \frac{\prod_{j=1}^p \Gamma(a_j + nA_j)}{\prod_{j=1}^q \Gamma(b_j + nB_j)} \frac{z^n}{n!}, \tag{1.10}$$

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