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ORIGINAL ARTICLE

On heat transfer analysis of axisymmetric flow of viscous fluid over a nonlinear radially stretching sheet

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Abstract This article deals with the exact solutions regarding heat transfer of viscous fluid over a non-linear radially stretching sheet. The effects of viscous dissipation on the heat transfer characteristics are considered in account in the presence of a transverse magnetic field for two types of boundary heating process namely prescribed power law surface temperature (PST) and prescribed heat flux (PHF). Similarity transformations are used to reduce the governing non-linear momentum and thermal boundary layer equations into a set of ordinary differential equations. The exact solutions of the reduced ordinary differential equations are developed in the form of confluent hypergeometric function and graphically sketched to see the influence of pertinent parameters on the temperature profiles. In addition the results for the wall temperature gradient (Nusselt number) are also presented in graphical form and discussed in detail. It is found that Prandtl number decelerates the temperature profiles in both PST and PHF cases. Thermal boundary layer thickness increases by increasing the Eckert number.

1. Introduction

The boundary layer flows and heat transfer mechanism over a stretching sheet have been investigated during the last few decades due to its significant applications in industrial and technological processes, for instance manufacturing of glass fiber, drawing plastic sheets, cooling and drying of papers, polymer melts. These processes highly depend on the heat transfer at the stretching surface. Crane [1] the pioneer of this work presented closed form exact solution of incompressible viscous fluid over planer stretching sheet. Wang [2] examined the exact solution of stretching problem into three dimensions. Alinejad and Samarbakhsh [3] examined the effects of viscous dissipation on viscous fluid flow over a non linearly stretching sheet. Sharma [4] studied the effects of viscous dissipation and heat source on unsteady boundary layer flow and heat transfer over a stretching sheet in a porous medium. Qasim [5] studied the combined effects of heat and mass transfer in a Jaffrey fluid over a stretching sheet and obtained the exact solution of the governing problem. Jat and Chand [6] analyzed the MHD flow and heat transfer over a exponentially stretching sheet with

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viscous dissipation. Cortell [7] reported the effects of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet. Heat and mass transfer of unsteady natural convection flow of some nanofluids past a vertical plate with radiation effect has been analyzed by Turkyilmazoglu [8]. Hamid and Ferdous [9] investigated the flow and heat transfer in a viscous nanofluid over a nonlinearly stretching sheet. Recently Ahmed et al. [10] obtained the exact solution for convective heat transfer in an MHD Jaffrey fluid over a stretching sheet with viscous dissipation in the presence of internal heat source and thermal radiations. Boundary layer flows and heat transfer over stretching surfaces have been investigated by many researchers [11–15] with various aspects.

Literature study infers that much more attention has been paid on heat transfer of fluid flow over a planner stretching; however, few studies regarding the radially stretching have been carried out. Sahoo [16] studied the influence of a partial slip on the axisymmetric flow of an electrically conducting viscoelastic fluid over a radially stretching sheet. Sajid et al. [17] presented the series solution for unsteady axisymmetric flow and heat transfer over a radially stretching sheet. Hayat et al. [18] obtained the homotopic solution for MHD axisymmetric flow of third grade fluid between stretching sheets with heat transfer. Shahzad et al. [19] examined the axisymmetric flow and heat transfer over a nonlinear radially stretching sheet for exact solution. Khan et al. [20] studied the MHD flow and heat transfer of a viscous fluid over a radially stretching power-law sheet with suction/injection in a porous medium. Ali et al. [21] investigated analytically as well as numerically the axisymmetric flow of viscous fluid with partial slip condition. Shateyi and Makinde [22] studied the hydromagnetic Stagnation point flow of a viscous fluid towards a radially stretching convectively heated disk by spectral relaxation method. Khan et al. [23] studied magnetohydrodynamic flow and heat transfer of Sisko fluid over a radially stretching sheet with convective boundary conditions analytically by HAM and numerically by shooting technique. Hayat et al. [24] considered the problem of MHD stagnation point flow of Jeffrey fluid by a radially stretching surface with viscous dissipation and joule heating.

Keeping in mind, the aim of this study was to investigate the MHD axisymmetric boundary layer flow and heat transfer over a nonlinear radially stretching sheet with viscous dissipation. Similarity transformations are used to convert the governing nonlinear partial differential equations into nonlinear ordinary differential equations. A closed form exact analytic solutions for the heat transfer are established in the form of confluent hypergeometric function for two general cases namely prescribed surface temperature PST case and prescribed heat flux PHF case. Graphical results for various values of governing physical parameters are revealed to gain thorough insight towards the physics of the problem. One can also find the solution of the governing nonlinear differential equations analytically [25–27].

2. Mathematical formulation

Consider the steady two-dimensional magnetohydrodynamic boundary layer flow of an electrically conducting, incompressible viscous fluid over a nonlinear radially stretching surface coinciding the plane $z = 0$. The flow is generated due to the stretching of the sheet along the radial direction with nonlinear velocity $U(r) = cr^3$, where c is a dimensional constant. It is assumed that the surface temperature of the sheet is T_w with an ambient fluid temperature T_∞, where $T_w > T_\infty$. Physical model under consideration is (see Fig. 1).

Under these assumptions boundary layer equations which govern the flow and heat transfer of a viscous fluid are [19]

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma B^2(r)}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \alpha\frac{\partial^2 T}{\partial z^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial z}\right)^2, \tag{3}$$

with u and w be the velocity components along the radial and axial directions, respectively, $\nu = \frac{\mu}{\rho}$ the kinematic viscosity, μ the coefficient of fluid viscosity, ρ the fluid density, T the fluid temperature, $\alpha = \frac{k}{\rho c_p}$ the thermal diffusivity, and $B(r) = B_o r$ is the variable magnetic field.

The corresponding boundary conditions for the momentum equation are

$$u = U(r) = cr^3, \quad w = 0 \text{ at } z = 0,$$
$$u \to 0 \text{ as } z \to \infty. \tag{4}$$

Now by introducing the following dimensionless transformations

$$u = U(r) = cr^3 f'(\eta), \qquad w = -\sqrt{c\nu}r[3f(\eta) + \eta f'(\eta)] \text{ and } \eta = \sqrt{\frac{c}{\nu}}rz. \tag{5}$$

Keeping in view Eq. (5), Eq. (1) is identically satisfied while the momentum Eq. (2) along with boundary conditions in Eq. (4) reduces to the following:

$$f''' + 3ff'' - 3f'^2 - Mf = 0, \tag{6}$$

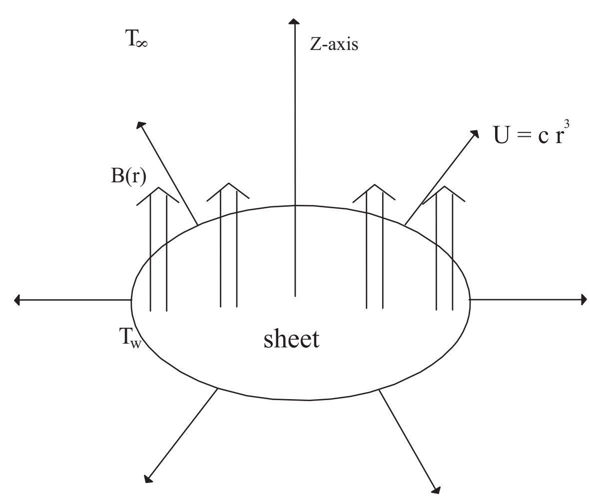

Figure 1 Schematic diagram of flow towards a radially stretching sheet.