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ORIGINAL ARTICLE

Fast calculation algorithm for discrete resonance-based band-pass filter

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Abstract A discrete resonant band-pass filter with a fast calculation algorithm, which can be used to perform discrete frequency transformations, is presented. The algorithm has low memory consumption requirements. It implements a numerical integration method, simulating a harmonic resonator element modeled by the under-damped driven oscillator equations, expressed in a discrete form. The output from the presented filter is a discrete function with an amplitude of the steady-solution that closely matches the theoretical steady-solution amplitude of the continuous band-pass filter output. Multiple discrete resonant band-pass filters can be used to build a filter bank, which in turn can be used to perform a time-to-frequency transformation of discrete signals. The filter achieves a frequency and a time localization without utilizing the time windowing method. The presented stand-alone calculation algorithm related to this filter produces its output with a delay of just one sampling period. The algorithm's calculation cost is only 3 multiplications and 3 additions per sample, and does not require long memory buffers. The presented transformation does not surpass the precision of the Discrete Fourier and Discrete Wavelet Transformations. However, it may prove essential when the noise-artifacts of the near-real-world simulation are necessary in order to produce some specific auditory-perception phenomena.

1. Introduction

Vast number of signal processing applications require frequency analysis of discrete (sampled) signals. Digital filters and discrete transformations into the frequency domain are used to fulfill those requirements.

Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filters are used to accomplish high-pass, low-pass, band-pass and other types of filtering. FIR filters are producing the output based only on the input signal, where the IIR filters are using past values of the output to produce the next output value (a feedback structure); therefore, they may show instability [1].

Best known FIR design method is the Generalized Window Method [2], while as usual IIR filter design methods are those derived from analog Butterworth, Chebyshev, and Elliptic Function filters [3]. Butterworth filters give maximally flat

Abbreviations: DRBF, discrete resonant band-pass filter; DRT, Discrete Resonant Transformation; DFT, Discrete Fourier Transformations; DWT, Discrete Wavelet Transformations; FIR, Finite Impulse Response; IIR, Infinite Impulse Response; BIBO, Bounded Input, Bounded Output (stability criterion).

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amplitude response; Chebyshev filters have a steeper roll-off and more passband or stopband ripples than Butterworth filters; and Elliptic Function filters have equalized ripple behavior between the passband and the stopband.

Diverse windowed Discrete Fourier Transformations (DFTs), Discrete Wavelet Transformations (DWTs) and other related transformations exist, addressing the transformation requirements. They transform sampled signals from one domain (time, location or other) into the frequency domain. The discrete transformations can be used as an integral part of a digital filter design. In addition, the filter banks can also be used to perform transformations into the frequency domain. Both transformation techniques, the discrete transformations and the filter banks, have advantages and disadvantages depending on the specifics of the application at hand [1]. In the following text, for simplicity, I utilize *time* as a source domain, but the reader should consider that the presented is equally applicable to other source domains.

This study was triggered by a research in the domain of computational neuroscience and neural networks, where near-real-world simulation was required in order to produce certain artifacts which may result in explanation of certain auditory perception phenomena. Windowed DFT and DWT produce artifacts that are not natural, but rather specific to the abstract mathematics that these transformations are based on.

Several constraints apply to DFT and DWT [4,5]. I present briefly those relevant to their comparison with the DRBF/DRT in this section. I disclose these constraints early, so they will serve as a context within which the DRBF/DRT methods are presented. In Section 5 these constraints are reviewed again, and some identified constraints of the DRBF/DRT are disclosed.

(1) DFT and DWT utilize time-windowing, introducing an error or noise within the result of the transformations, due to the harmonic components caused by the time-window boundaries.
(2) DFT and DWT introduce delay in production of the transformation result, caused by the need to collect a buffer of samples. Wider buffer (bigger number of samples in the time window) is required in order to achieve better frequency precision. Consequently, attempting to increase the frequency precision, DFT and DWT increase the delay, constraining their applicability in real-time applications.
(3) In order for DFT and DWT calculation algorithms to perform efficiently, the number of the samples in the buffer must be a power of 2 (2^n). This further constrains the ability to achieve finer compromise between the time localization and frequency precision.
(4) The calculation complexity (cost) achieved so far is O (NLog(N)) for DFT (per buffer of N samples) and O (N) for DWT (per scale) [6]. DFT and DWT implemented algorithms require significant amount of computing memory used to maintain the buffers, additionally decreasing the performance due to operations such as memory reallocation, retrieving and assigning values.

Here I propose a discrete resonant band-pass filter (DRBF) fast calculation algorithm. The DRBF can be used to perform fast time-to-frequency domain transformations, taking an approach different than the DFT and DWT calculation methods. For the purpose of this text I will name this transformation as Discrete Resonant Transformation (DRT). The DRT overcomes some of the outlined constraints of the DFT and DWT. I will occasionally use the term *resonator* to refer to the DRBF when stressing the resonance events is valuable. As of the time of writing this document, a benchmark comparison between the DRT and the DFT/DWT implementations has not been performed.

2. Mathematical foundation

We start with a system, consisting of a harmonic *resonator* that oscillates according to the well-known equation of a driven harmonic oscillator ([6], pp. 211):

$$m\frac{d^2x}{dt^2} = -c\frac{dx}{dt} - kx + D(t) \tag{1}$$

where
- m is the mass that oscillates
- x is the elongation of the oscillator (the distance from the stable position at time t)
- t is the time dimension
- c is a viscous damping coefficient; it determines the deceleration of the oscillations due to a friction, which in turn depends on the current speed of the oscillator ($\frac{dx}{dt}$)
- k is a coefficient that determines the magnitude of the force that pulls the oscillator back to its stable position (stiffness), which depends on the elongation
- $D(t)$ is the driving force, represented as a function of time.

Eq. (1) describes that the acceleration of the oscillating mass is caused by the following:

- the driving force at the current moment of time,
- the distance of the center of the oscillating mass from its stable position at the present moment of time, and
- the speed of the oscillating mass at the current moment of time.

Converting (1) from a continuous to a discrete form, inspired by the Störmer–Verlet integration method [7], assuming that the sampling rate is much higher than the oscillating frequency, we get:

$$m\frac{\Delta^2 x_i}{\Delta t^2} = -c\frac{\Delta x_{i-1}}{\Delta t} - kx_{i-1} + D_{i-1} \tag{2}$$

where
- Δt is the sampling period of time (note: the sampling rate is considered constant and it must be much higher than the resonant frequency – see Section 5)
- $\frac{\Delta x_{i-1}}{\Delta t}$ is the speed (v_{i-1}), at the moment $(i-1)\Delta t$
- $\frac{\Delta^2 x_i}{\Delta t^2}$ is the acceleration at the moment $i\Delta t$
- D_{i-1} is the sample of the driving force's magnitude at the moment $(i-1)\Delta t$.

This conversion introduces an error that will be analyzed in Appendix C. If we express the speed (v) and the acceleration (a) as follows: