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ORIGINAL ARTICLE

A nonlinear model for fully fuzzy linear programming with fully unrestricted variables and parameters

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Abstract Several methods currently exist for solving fuzzy linear programming problems under nonnegative fuzzy variables and restricted fuzzy coefficients. However, due to the limitation of these methods, they cannot be applied for solving fully fuzzy linear programming (FFLP) with unrestricted fuzzy coefficients and fuzzy variables. In this paper a new efficient method for FFLP has been proposed, in order to obtain the fuzzy optimal solution with unrestricted variables and parameters. This proposed method is based on crisp nonlinear programming and has a simple structure. To show the efficiency of our proposed method some numerical examples have been illustrated.

1. Introduction

Linear Programming (LP) is one of the most important techniques in operational research. Many real-world problems can be transformed into a linear programming model. Hence this model is an indispensable tool for today's applications, such as energy, transportation and manufacturing. In real world applications, certainty, reliability and precision of data are often illusory. The optimal solution of an LP only depends on a limited number of constraints; therefore, much of the collected information has little impact on the solution. It is useful to consider the knowledge of experts about the parameters as fuzzy data. The concept of fuzzy set was introduced in [1]. After this, Bellman and Zadeh [2] introduced fuzzy optimization problems where they have stated that, a fuzzy decision can be viewed as the intersection of fuzzy goals and problem constraints. Many researchers such as Zimmermann [3], Tanaka and Asai [4], Campos and Verdegay [5], Rommelfanger et al. [6], Cadenas and Verdegay [7] who were dealing with the concept of solving fuzzy optimization problems, later studied this subject. In the past few years, a growing interest has been shown in fuzzy optimization. Buckley and Feuring [8] introduced a general class of fuzzy linear programming, called fully fuzzified linear programming problems, where all decision parameters and variables are fuzzy numbers. Lodwick and Bachman [9] have studied large scale fuzzy and possible optimization problems. Buckley and Abdalla [10] have considered Monte Carlo methods in fuzzy queuing theory. Some authors have considered fuzzy linear programming, in which

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not all parts of the problem were assumed to be fuzzy, e.g., only the right hand side and the objective function coefficients were fuzzy, or only the variables were fuzzy [11–17]. The fuzzy linear programming problems in which all the parameters and variables are represented by fuzzy numbers are known as fully fuzzy linear programming (FFLP) problems. The FFLP problem with inequality constraints was studied in [8,18,19]. However, the main disadvantage of the solution obtained by the existing methods is that, it does not satisfy the constraints exactly i.e. it is not possible to obtain the fuzzy number of the right hand side of the constraint by putting the obtained solution on the left hand side of the constraint.

Dehghan et al. [20] proposed some practical methods to solve the fully fuzzy linear system (FFLS) that is comparable to the well known methods. Then they extended a new method employing Linear Programming (LP) for solving square and non-square fuzzy systems. Lotfi et al. [21] applied the concept of the symmetric triangular fuzzy number, and obtained a new method for solving FFLP by converting a FFLP into two corresponding LPs. Kumar et al. [22] pointed out the shortcomings of the above methods. To overcome these shortcomings, they proposed a new method for finding the fuzzy optimal solution of FFLP problems with equality constraints. Saberi Najafi and Edalatpanah [23] pointed out the method of [22] needs some corrections to make the model well in general. Kaur and Kumar [24] proposed a new method for FFLP problems with equality constraints having non-negative fuzzy coefficients and unrestricted fuzzy variables. Jayalakshmi and Pandian [25] presented a new method where, the given FFLP problem is decomposed into three crisp linear programming (CLP) problems with bounded variable constraints. Fan et al. [26] adopted the α-cut level to deal with a generalized fuzzy linear programming problem. Shamooshaki et al. [27] using L-R fuzzy numbers and ranking function established a new scheme for FFLP. Ezzati et al. [28] using a new ordering on triangular fuzzy numbers and converting FFLP to a multi-objective linear programming (MOLP) problem, presented a new method to solve FFLP. Recently, a number of researchers have exhibited their interest to solve the fuzzy linear programming problems [29–39]. In general, the above existing methods can be applied to the following types of FFLP problems:

(i) FFLP problem with nonnegative fuzzy coefficients and nonnegative fuzzy variables.
(ii) FFLP problem with unrestricted fuzzy coefficients and nonnegative fuzzy variables.
(iii) FFLP problem with nonnegative fuzzy coefficients and unrestricted fuzzy variables.

In crisp LP topic and application of it, we can see some LP problem in which some or all the variables are unrestricted. So these situations can be occurring in fuzzy linear programming problem, i.e. a fuzzy linear programming problem in which some or all the parameters are represented by unrestricted fuzzy numbers. As far as we are aware, there are only a few papers dealing with solution of fuzzy LP problems with fully unrestricted variables and parameters. In this paper, we focus on this problem and propose a new efficient method for solving FFLP with unrestricted variables and parameters. The proposed method is based on crisp nonlinear programming and has a simple and clear structure.

2. Preliminaries

In this section, we give some basic notations and preliminary results which are essential tools for describing our main results. For details, we refer to [22–24].

Definition 2.1. Let X denote a universal set. Then a fuzzy subset $\widetilde{A}$ of X is defined by its membership function $\mu_{\widetilde{A}}: X \to [0,1]$; which assigns a real number $\mu_{\widetilde{A}}(x)$ in the interval $[0,1]$, to each element $x \in X$, where the value of $\mu_{\widetilde{A}}(x)$ at x shows the grade of membership of x in $\widetilde{A}$.

A fuzzy subset $\widetilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\widetilde{A}}(x)$ and is often written $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x));\ x \in X\}$.

The class of fuzzy sets on X is denoted with $\Gamma(X)$.

Definition 2.2. A fuzzy number $\widetilde{A} = (a,b,c)$ is said to be a triangular fuzzy number if its membership function is given

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leqslant x \leqslant b, \\ \frac{(c-x)}{(b-c)}, & b \leqslant x \leqslant c, \\ 0, & \text{else.} \end{cases}$$

Definition 2.3. A triangular fuzzy number $\widetilde{A} = (a,b,c)$ is said to be nonnegative fuzzy number if and only if $a \geqslant 0$. The set of non-negative fuzzy numbers may be represented by $F(R^+)$.

Definition 2.4. A triangular fuzzy number $\widetilde{A} = (a,b,c)$ is said to be unrestricted fuzzy number if $a,b,c \in R$. The set of unrestricted fuzzy numbers can be represented by $F(R)$.

Definition 2.5. Let $\widetilde{A} = (a,b,c)$, $\widetilde{B} = (d,e,f)$ be two triangular fuzzy numbers then;

(i) $\widetilde{A} \oplus \widetilde{B} = (a,b,c) \oplus (d,e,f) = (a+d, b+e, c+f)$,
(ii) $\widetilde{A} - \widetilde{B} = (a,b,c) - (d,e,f) = (a-f, b-e, c-d)$,
(iii) $\widetilde{A} \otimes \widetilde{B} = (\min(\gamma), be, \max(\gamma))$ where, $\gamma = \{ad, af, cd, cf\}$.

Definition 2.6. Two triangular fuzzy numbers $\widetilde{A} = (a,b,c)$, $\widetilde{B} = (d,e,f)$ are said to be equal if and only if $a = d$, $b = e$ and $c = f$.

Definition 2.7. A ranking function is a function $R: F(R) \to R$, which maps each fuzzy number into the real line, where a natural order exists. Let $\widetilde{A} = (a,b,c)$ be any triangular fuzzy number, then $R(\widetilde{A}) = \frac{1}{4}(a + 2b + c)$.

Definition 2.8. Let $\widetilde{A} = (a,b,c)$, $\widetilde{B} = (d,e,f)$ be two triangular fuzzy numbers, then

(i) $\widetilde{A} \leqslant \widetilde{B}$ iff $R(\widetilde{A}) \leqslant R(\widetilde{B})$,
(ii) $\widetilde{A} < \widetilde{B}$ iff $R(\widetilde{A}) < R(\widetilde{B})$.

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