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ORIGINAL ARTICLE

Steady state response of functionally graded nano-beams resting on viscous foundation to super-harmonic excitation

Sima Ziaee

Department of Mechanical Engineering, Yasouj University, Yasouj, Iran

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Abstract This article attempts to investigate the effects of small scale parameter on steady state response of functionally graded nano-beams resting on a viscous foundation to super-harmonic excitation. A simple power-law distribution is used to model the variation of material property graded in the thickness direction. The dimensionless partial differential equation of motion is derived by using Euler-Bernoulli beam theory, von-Karman geometric nonlinearity and Eringen's nonlocal elasticity theory. Using multiple scale method, one can find the governing equations of steady state response of functionally graded nano-beams excited by distributed harmonic force. The small scale parameter (e_0a) is changed between 0 and 2 to investigate the effects of small scale on steady state response of excited functionally graded nano-beams due to lack of information. The study of the effects of small scale parameter on backbone curves shows that an increase in the small scale parameter often decreases the dimensionless peak response although the type of loading can change the relationship between small scale parameter and the dimensionless peak response.

1. Introduction

Functionally graded materials (FGM) have unique characteristics resulting from the smooth and continuous variation of properties along certain dimensions. These remarkable advantages have made them the notable materials which can be used in many engineering application fields [1]. The growing development of technology has permitted the use of FGM thin beams in micro/nano-electro-mechanical systems, such as electrically actuated devices and atomic force microscopes [2], so the study of mechanical behavior of FG micro-/nano-beams has recently become a topic of interest to researchers.

Using a modified couple stress theory, Reddy [3] derived the nonlinear non-classical Timoshenko and Euler-Bernoulli beam theories to study static bending, free vibration, and buckling of FG simply supported micro-beams. These theories were used by Arbind and Reddy [4] to investigate nonlinear bending response of clamped FG micro-beams under mechanical loadings while Eltaher et al. [5,6] and Şimşek and Yurtcu [7] independently employed nonlocal Timoshenko beam theory [5,7] and nonlocal Euler-Bernoulli beam theory [6,7] to study the static bending and buckling of FG nano-beams with different boundary conditions. Some researchers used the

E-mail addresses: simaziaee@gmail.com, sima_ziaee@yahoo.com, Ziaee@yu.ac.ir

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combination of surface elasticity theory [8,9] or the strain gradient theories [10–12] with nonlinear beam theories to study nonlinear free vibration of functionally graded nano-beams as well.

All analytical and numerical results reported by researchers mentioned above clearly show the difference between classical and non-classical continuum theories in predicting mechanical behavior of size-dependent FG beams. Due to the necessity of incorporating the size effect into classical continuum mechanics to study mechanical behavior of FG micro-/nano-beams accurately, the nonlocal elasticity theory has been employed to simulate dynamical behavior of micro/nano-beams by some researchers.

Eltaher et al. [13] studied free vibration of FG nano-beams based upon nonlocal Euler-Bernoulli beam theory and finite element method. The effects of neutral axis location on linear natural frequencies of FG macro-/nano-beams were investigated by Eltaher et al. [14] as well. Uymaz [15] used generalized beam theory and the nonlocal elasticity to present forced vibration of FG nano-beams. Nonlinear free vibration of FG nano-beams was studied by Nazemnezhad and Hosseini-Hashemi [16] based on nonlocal Euler-Bernoulli beam theory and multiple scale method. Using nonlocal Timoshenko beam theory, Rahmani and Pedram [17] investigated the effects of gradient index and geometrical dimensions on linear free vibration of FG nano-beams. He's variational method and nonlocal Euler-Bernoulli beam theory were used to study the large amplitude free vibration of FG nano-beams resting on nonlinear elastic foundation by Niknam and Aghdam [18]. Kiani [19] proposed a mathematical model to investigate the vibration and instability of moving FG nano-beams based on nonlocal Rayleigh beam theory. Obtained results clearly show that the value of small scale parameter is an important factor to estimate dynamic responses of FG nano-beams accurately although boundary conditions, order of the mode of vibration and geometrical dimensions can affect the role of the small scale parameter in simulating dynamic responses of FG nano-beams.

Although it is well known that proper values of the nonlocal parameter must be used if the accurate study of mechanical behavior of micro-/nano-structures is desired, a thorough research has not been done to estimate the value of small scale corresponding to mechanical response of functionally graded micro-/nano-beams so far [16]. Hence, all researchers who used nonlocal continuum theories to simulate size-dependent mechanical behavior of FG nano-beams investigated the effects of small scale parameter on mechanical behavior of FG nano-beams by changing the value of the small scale parameter [13–16].

Based upon the author's knowledge, there is no notable study showing the influence of nonlocal parameter on mechanical response of FG nano-beams to sub- or super-harmonic excitation. So, the investigation of the effects of small scale parameter on steady-state response of FG nano-beams resting on a viscous foundation to super-harmonic excitation is the main purpose of this article. A simple power-law distribution is used to model the variation of material property graded in the thickness direction. The partial differential equation of motion is derived based on Euler-Bernoulli beam theory, von-Karman geometric nonlinearity and Eringen's nonlocal elasticity theory. The multiple scale method is employed to find governing equations of steady state response of FG nano-beams excited by distributed harmonic force. In the parametric studies of this work, due to lack of information, small scale (e_0a) is varied between 0 and 2 to investigate the effects of small scale on steady state response of excited FG nano-beams.

2. Equation of motion

The following equation is the partial differential equation of motion of a simply supported FG nano-beam resting on viscous foundation (Fig. 1) with length L, width b and thickness h and immovable ends. This equation is derived based on Euler-Bernoulli beam theory hypothesis, von-Karman geometric nonlinearity and Eringen's nonlocal elasticity theory (details can be found in Appendix A):

$$\begin{aligned}&(e_0a)^2H+\widehat{N}\frac{\partial^2W}{\partial x^2}-\hat{k}W-\hat{c}\frac{\partial W}{\partial t}+F(x,t)\\&=\left(\int_{A_0}(z-z_0)^2E(z)dA_0\right)\frac{\partial^4W}{\partial x^4}+\left(\int_{A_0}\rho(z)dA_0\right)\frac{\partial^2W}{\partial t^2}\end{aligned}\quad(1)$$

where H is defined by Eq. (2):

$$H=\hat{c}\frac{\partial^3W}{\partial x^2\partial t}+\hat{k}\frac{\partial^2W}{\partial x^2}-\frac{\partial^2F}{\partial x^2}-\widehat{N}\frac{\partial^4W}{\partial x^4}+\left(\int_{A_0}\rho(z)dA_0\right)\frac{\partial^4W}{\partial x^2\partial t^2}\quad(2)$$

and $\widehat{N}$ is as follows:

$$\widehat{N}=+\frac{1}{2L}\left(\int_{A_0}E(z)dA_0\right)\int_0^L\left(\frac{\partial W}{\partial x}\right)^2dx\quad(3)$$

where $W=W(x,t)$ denotes the transverse displacement of any point on the geometric mid-plane ($z=0$, based on Fig. 1) of beam element, $\rho(z)$ is mass density which is functionally graded in the thickness direction, $\hat{k}$ and $\hat{c}$ are the stiffness of the foundation and the damping coefficient of the foundation respectively, $F=F(x)\cos(\Omega t)$ is a transverse loading, and e_0a is a material length scale parameter which contains material constant and internal characteristic length. The distance of the neutral surface of the FG nano-beam from the geometric mid-plane of the FG nano-beam ($z=0$, based on Fig. 1) is shown by z_0 [14].

One can employ the following dimensionless variables to simplify the parametric studies [20]:

$$\begin{aligned}&\bar{x}=\frac{x}{L},\ \overline{W}=\frac{W}{r},\ \bar{t}=t\sqrt{D/\rho_eL^4},\\&D=b\int_{-\frac{h}{2}}^{\frac{h}{2}}z^2E(z)dz,\rho_e=b\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho(z)dz,\ A=b\int_{-\frac{h}{2}}^{\frac{h}{2}}E(z)dz,\\&r=\sqrt{bh^3/12(bh)}\end{aligned}\quad(4)$$

Then, the governing partial deferential equation of motion changes to the following:

$$\begin{aligned}&\frac{-L^4(e_0a)^2}{rD}\overline{H}+\frac{\partial^2\overline{W}}{\partial\bar{t}^2}+\frac{\partial^4\overline{W}}{\partial\bar{x}^4}+\frac{\hat{k}L^4}{D}\overline{W}\\&+\frac{\hat{c}L^2}{\sqrt{D\rho_e}}\frac{\partial\overline{W}}{\partial\bar{t}}-\left(\frac{Ar^2}{2D}\int_0^1\left(\frac{\partial\overline{W}}{\partial\bar{x}}\right)^2d\bar{x}\right)\frac{\partial^2\overline{W}}{\partial\bar{x}^2}=F\frac{L^4}{rD}\end{aligned}\quad(5a)$$