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ORIGINAL ARTICLE

New wavelet based full-approximation scheme for the numerical solution of nonlinear elliptic partial differential equations

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Abstract Recently, wavelet analysis application has dragged the attention of researchers in a wide variety of practical problems, particularly for the numerical solution of nonlinear partial differential equations. Based on Daubechies filter coefficients, a modified method using wavelet intergrid operators known as new wavelet based full-approximation scheme (NWFAS) similar to multigrid full-approximation scheme (FAS) is developed for the numerical solution of nonlinear elliptic partial differential equations. The present method gives higher accuracy in terms of better convergence with low CPU time. The results of tested examples of proposed method show better performance which is demonstrated through the illustrative examples.

1. Introduction

Nonlinearity is essential in the majority of physical phenomena and engineering processes, resulting in nonlinear differential equations. In fact, these nonlinear equations are usually difficult to solve, since no general technique works globally. Hence each individual equation has to be studied as a separate problem. In the particular interest, the nonlinear elliptic type equations are encountered in transport problems, notably fluid flow problems. Solutions to this type of problems are usually required more CPU time with slow convergence. The nonlinear character of the partial differential equations that govern these problems reduces the analytical solution is difficult. In fact, the classical methods (for example finite difference method) are used to solve the problems with low accuracy in more CPU time.

System of nonlinear equations is difficult to solve in general, usually, solving these equations using iterative methods such as Newton's method, Jacobi iterative method, and Gauss-Seidel method. The FAS is largely applicable in increasing the efficiency of the iterative methods used to solve nonlinear system of algebraic equations. FAS is a well-founded numerical method for solving nonlinear system of equations for a approximating given differential equation. In the historical three decades the development of effective iterative solvers for nonlinear systems of algebraic equations has been a significant research topic in numerical analysis, computational science and engineering. Nowadays it is recognized that FAS iterative solvers are highly efficient for nonlinear differential

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equations introduced by Brandt [1]. A detailed treatment of FAS is given in Briggs et al. [2]. An introduction of FAS is found in Hackbusch and Trottenberg [3], Wesseling [4] and Trottenberg et al. [5]. Many authors namely, Brandt [1] and Briggs et al. [2], applied the FAS to some classes of differential equations. Lubrecht [6], Venner and Lubrecht [7], Zargari et al. [8] and others have significant contributions in elasto-hydrodynamic lubrication (EHL) problems.

Wavelets have several applications in approximation theory and have been widely used in the numerical approximation since from more than two decades. Wavelet based numerical methods are used for solving the system of equations with faster convergence and low computational cost. Some of the earlier works can be found in Dahmen et al. [9]. The many numerical methods employ various types of wavelets for the numerical solutions of different classes of differential equations; some of them are Haar wavelets by Bujurke et al. [10–12] and Islam et al. [13], Ebolian and Fattahzdeh [14] used Chebyshev wavelets, B-Spline by Dehghan and Lakestani [15]. Due to desirable properties, researchers are now paying attention to Daubechies wavelet. Recently, Daubechies compactly supported orthogonal wavelet is widely used. Diaz et al. [16], Avudainayagam and Vani [17] and Bujurke et al. [18–20] and many researchers applied Daubechies wavelet method to some classes of differential equations. The main aim of the present paper was to introduce the new wavelet based FAS for the solution of nonlinear elliptic partial differential equations arising in science and engineering.

The organization of this paper is as follows: Daubechies wavelet filter coefficients are given in Section 2. Section 3, presents the method of solution and intergrid operators. Numerical findings of the test problems are presented in Section 4. Finally, conclusions of the proposed work are discussed in Section 5.

2. Daubechies wavelet filter coefficients

The class of compactly supported wavelet bases was introduced by Daubechies [21]. They are an orthonormal bases for functions in $L_2(R)$. A family of orthogonal Daubechies wavelets with compact support has been constructed by [22]. Due to excellent properties of orthogonality and minimum compact support, Daubechies wavelets can be useful and convenient, providing guaranty of convergence and accuracy of the approximation in a wide variety of situations.

In this paper, we use Daubechies filter coefficients for $N = 4$ which are

Low pass filter coefficients: $h_0 = \frac{1+\sqrt{3}}{4\sqrt{2}}$, $h_1 = \frac{3+\sqrt{3}}{4\sqrt{2}}$, $h_2 = \frac{3-\sqrt{3}}{4\sqrt{2}}$, $h_3 = \frac{1-\sqrt{3}}{4\sqrt{2}}$ and

High pass filter coefficients: $g_0 = \frac{1-\sqrt{3}}{4\sqrt{2}}$, $g_1 = -\frac{3-\sqrt{3}}{4\sqrt{2}}$, $g_2 = \frac{3+\sqrt{3}}{4\sqrt{2}}$, $g_3 = -\frac{1+\sqrt{3}}{4\sqrt{2}}$.

2.1. Discrete wavelet transform (DWT) matrix

The matrix formulation of the discrete wavelet transforms (DWT) plays an important role in the wavelet method for the numerical computations. As we already know about the DWT matrix and its applications in the wavelet method and is given in [22] as,

$$W_1 = \begin{pmatrix} h_0 & h_1 & h_2 & h_3 & 0 & 0 & \dots & & 0 & 0 \\ g_0 & g_1 & g_2 & g_3 & 0 & 0 & \dots & & 0 & 0 \\ 0 & 0 & h_0 & h_1 & h_2 & h_3 & \dots & & 0 & 0 \\ 0 & 0 & g_0 & g_1 & g_2 & g_3 & \dots & & 0 & 0 \\ \vdots & & \ddots & & \dots & & \dots & & 0 & 0 \\ h_2 & h_3 & 0 & 0 & \dots & & \dots & 0 & h_0 & h_1 \\ g_2 & g_3 & 0 & 0 & \dots & & \dots & 0 & g_0 & g_1 \end{pmatrix}_{N\times N}$$

Using this matrix authors used restriction and prolongation operators W and W^T respectively given in Section 3.2.

2.2. New discrete wavelet transform (NDWT) matrix

Here, we developed NDWT matrix similar to DWT matrix in which by adding rows and columns consecutively with diagonal element as 1, which is built as,

$$W_2 = \begin{pmatrix} h_0 & 0 & h_1 & 0 & h_2 & 0 & h_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \dots & & & \dots & 0 & 0 \\ g_0 & 0 & g_1 & 0 & g_2 & 0 & g_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & & \dots & 0 & 0 \\ \vdots & & \ddots & & \dots & & \dots & & & & 0 & 0 \\ g_2 & 0 & g_3 & 0 & \dots & & \dots & 0 & g_0 & 0 & g_1 & 0 \\ 0 & 0 & \dots & & \dots & & \dots & & & \dots & 0 & 1 \end{pmatrix}_{N\times N}$$

Using W_2 matrix, we introduced restriction and prolongation operators M and M^T respectively such as wavelet multigrid operators given in Section 3.3.

3. Method of solution

Consider the nonlinear partial differential equation. After discretizing the partial differential equation through the FDM, we get the system of nonlinear equations of the form,

$$F(u_{ij}) = b_{ij}, \tag{1}$$

where $i,j = 1,2,\dots,N$, which have $N \times N$ equations with $N \times N$ unknowns. Solve Eq. (1) through iterative method Gauss Seidel (GS), we get approximate solution v. Approximate solution contains some errors, and therefore required solution equals to sum of approximate solution and error. There are many methods to minimize such error to get the accurate solution. Some of them are FAS, WFAS, NWFAS, etc. Now we are discussing the method of solution of the above mentioned methods as below.

3.1. Full-Approximation Scheme (FAS)

The algorithm of FAS given by Briggs et al. [2] is as follows,

Step – 1:

From the system (1), we get the approximate solution v for u. Now we find the residual as

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