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ORIGINAL ARTICLE

Chaikin's perturbation subdivision scheme in non-stationary forms

Wardat us Salam [a,*], **Shahid S. Siddiqi** [b], **Kashif Rehan** [c]

[a] *Department of Mathematics, University of the Punjab, Quaid-e-Azam Campus, Lahore 54590, Pakistan*
[b] *Abdus Salam School of Mathematical Sciences, GC University, Lahore, Pakistan*
[c] *Department of Mathematics, University of Engineering and Technology, KSK Campus, Lahore, Pakistan*

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Abstract In this paper two non-stationary forms of Chaikin's perturbation subdivision scheme, mentioned in Dyn et al. (2004), have been proposed with tension parameter ω. Comparison among the proposed subdivision schemes and the existing non-stationary subdivision scheme depicts that the trigonometric form is more efficient in the reproduction of circles and ellipses and the hyperbolic form is more suitable for the construction of many analytical curves.

1. Introduction

Subdivision schemes ascend from modeling and interrogation of curves and surfaces, image reconstruction and decomposition, and the construction problems of compact supported wavelet. These schemes are being developed in geometric modeling with great potentiality in computer graphics, CAM/CAD and image processing. Subdivision schemes are widely used in garment CAD, jewelry CAD and computer graphics industry. These schemes are also important in fractal generation by computer particularly [2,3]. Subdivision schemes are used to construct the required curves and surfaces from scattered data directly through stated subdivision rules. Mask of the subdivision schemes is simply averaging rules corresponding to odd and even subsequences of finitely supported sequence of real numbers. In case of level dependent subdivision schemes [4–9] mask varies from one level to another; generally, it allows to generate larger variety of limiting curves having several useful properties *e.g.* reproduction of conics and spirals etc. New methods of convergence of non-stationary schemes have been introduced in [18,19]. In [18] asymptotic similarity has been used instead of asymptotic equivalence. In [19] spectral radius approach has been used along with the asymptotic similarity for convergence. Different properties of the non-stationary subdivision schemes *e.g.* approximation order and reproduction properties have been analyzed in [20–23].

Some numerical schemes have been presented by P. Das and S. Natesan to solve singularly perturbed reaction diffusion differential equations in [15–17].

In this paper, Chaikin's perturbation subdivision scheme [1] has been presented in trigonometric and hyperbolic forms with the abilities to reproduce conic-sections and many analytical curves.

* Corresponding author.
E-mail addresses: wardahtussalam@gmail.com (W.us Salam), shahidsiddiqiprof@yahoo.co.uk (S.S. Siddiqi), kkashif_99@yahoo.com (K. Rehan).
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The following basic results of the non-stationary subdivision schemes are considered to prove the convergence/ smoothness of the proposed non-stationary subdivision schemes.

Definition 1.1. Given a set of initial control points $P^0 = \{p_i^0 \in R^d\}_{i=-1}^{k+1}$, a binary subdivision scheme generates new set of control points $P^n = \{p_i^n\}_{i=-1}^{2^n k+1}$ at level $n (n \geqslant 0, k \in Z)$ by the subdivision rule

$$p_i^{n+1} = \sum_{j \in z} a_{i-2j}^n p_j^n, \quad i \in Z,$$

where the set of coefficients $a^{(n)} = \{a_i^{(n)}, i \in Z\}$ in above equation is termed as the mask of the subdivision scheme at n^{th} subdivision step. The Laurent polynomial associated with the non-stationary subdivision scheme $\{S_{a^n}\}$ having mask $a^{(n)}$ is

$$a^n(z) = \sum_{i \in Z} a_i^{(n)} z^i, \quad n \geqslant 1.$$

Definition 1.2 [10]. A binary subdivision scheme $\{S_{a^n}\}$ is said to be C^m if for every initial data $p^0 = \{p_i^0 : i \in Z\}$ there exists a limit function $f \in C^m$ such that for any closed interval $K = [a, b] \subset \mathbb{R}$,

$$\lim_{n \to \infty} \max_{i \in \mathbb{Z} \bigcap 2^n K} |p_i^n - f(2^{-n} i)| = 0.$$

Obviously $f = S^\infty p^0 \neq 0$ for some initial data p^0 and also p_i^n are the control points at level n.

Definition 1.3 [10]. Two binary subdivision schemes $\{S_{a^n}\}$ and $\{S_{b^n}\}$ are asymptotically equivalent if

$$\sum_{n=1}^{\infty} \|S_{a^n} - S_{b^n}\| < \infty,$$

where $\|S_{a^n}\|_\infty = \max\{\sum_{i \in Z} |a_{2i}^{(n)}|, \sum_{i \in Z} |a_{2i+1}^{(n)}|\}$.

Theorem 1.1 [11]. *Let $\{S_{a^n}\}$ and $\{S_a\}$ be the two asymptotically equivalent subdivision schemes having finite masks of the same support. Suppose $\{S_{a^n}\}$ is a level dependent subdivision scheme and $\{S_a\}$ is a stationary subdivision scheme. If $\{S_a\}$ is C^m and*

$$\sum_{n=0}^{\infty} 2^{mn} \|S_{a^n} - S_a\| < \infty,$$

then the non-stationary subdivision scheme $\{S_{a^n}\}$ is C^m.

The organization of paper is as follows. In Section 2, Chaikin's perturbation (binary four point approximating) subdivision scheme has been recalled. In Section 3, the trigonometric and hyperbolic forms of Chaikin's perturbation subdivision scheme have been presented. Convergence analysis of the proposed schemes has also been discussed in Section 3. The normalization of the proposed schemes has been given in Section 4 as these schemes do not observe the affine invariance property. In Section 5, some properties of the proposed schemes have been discussed. Graphical behavior of the proposed schemes has been exhibited along with their comparison in Section 6.

2. Chaikin's perturbation subdivision scheme

Given a set of control points $f^0 = \{f_i^0\}_{i \in \mathbb{Z}}$ at level 0, Chaikin's perturbation subdivision scheme [1] generates a new set of control points $\{f_i^n\}_{i \in \mathbb{Z}}$ at the level $n+1$ by applying the following subdivision rules:

$$\begin{cases} f_{2i}^{n+1} = -7\omega f_{i-1}^n + \left(\frac{3}{4} + 9\omega\right) f_i^n + \left(\frac{1}{4} + 3\omega\right) f_{i+1}^n - 5\omega f_{i+2}^n, \\ f_{2i+1}^{n+1} = -5\omega f_{i-1}^n + \left(\frac{1}{4} + 3\omega\right) f_i^n + \left(\frac{3}{4} + 9\omega\right) f_{i+1}^n - 7\omega f_{i+2}^n, \end{cases} \quad (1)$$

with $\omega = 0$ corresponds to the Chaikin's scheme [12]. The scheme gives C^1- continuous limit curves for $\omega = 0$ and C^2- continuous limit curves for $0 < \omega < \frac{\sqrt{6}-1}{80}$.

3. Non-stationary schemes for uniform trigonometric and hyperbolic spline curves

In this section, trigonometric and hyperbolic forms of Chaikin's perturbation subdivision scheme [1] have been presented.

3.1. Trigonometric form

The four point non-stationary subdivision scheme is

$$\begin{cases} f_{2i}^{n+1} = \beta_0^n f_{i-1}^n + \beta_1^n f_i^n + \beta_2^n f_{i+1}^n + \beta_3^n f_{i+2}^n, \\ f_{2i+1}^{n+1} = \beta_3^n f_{i-1}^n + \beta_2^n f_i^n + \beta_1^n f_{i+1}^n + \beta_0^n f_{i+2}^n, \end{cases} \quad (2)$$

where

$$\beta_0^n = -7\omega,$$

$$\beta_1^n = \frac{s\left(\frac{3\alpha}{2^{n+2}}\right)}{s\left(\frac{\alpha}{2^n}\right)} + 9\omega,$$

$$\beta_2^n = \frac{s\left(\frac{\alpha}{2^{n+2}}\right)}{s\left(\frac{\alpha}{2^n}\right)} + 3\omega,$$

$$\beta_3^n = -5\omega,$$

where $s(t) = \sin(t)$ and $c(t) = \cos(t)$.

3.2. Hyperbolic form

The four point non-stationary subdivision scheme is

$$\begin{cases} f_{2i}^{n+1} = \gamma_0^n f_{i-1}^n + \gamma_1^n f_i^n + \gamma_2^n f_{i+1}^n + \gamma_3^n f_{i+2}^n, \\ f_{2i+1}^{n+1} = \gamma_3^n f_{i-1}^n + \gamma_2^n f_i^n + \gamma_1^n f_{i+1}^n + \gamma_0^n f_{i+2}^n, \end{cases} \quad (3)$$

where

$$\gamma_0^n = -7\omega,$$

$$\gamma_1^n = \frac{s'\left(\frac{3\alpha}{2^{n+2}}\right)}{s'\left(\frac{\alpha}{2^n}\right)} + 9\omega,$$

$$\gamma_2^n = \frac{s'\left(\frac{\alpha}{2^{n+2}}\right)}{s'\left(\frac{\alpha}{2^n}\right)} + 3\omega,$$

$$\gamma_3^n = -5\omega,$$

where $s'(t) = \sinh(t)$ and $c'(t) = \cosh(t)$.

3.3. Continuity analysis

Asymptotic equivalence is needed to be established for continuity analysis.