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Interactive Approach for Multi-Level Multi-Objective Fractional Programming Problems with Fuzzy Parameters

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ABSTRACT

In this paper, an interactive approach for solving multi-level multi-objective fractional programming (ML-MOFP) problems with fuzzy parameters is presented. The proposed interactive approach makes an extended work of Shi and Xia (1997). In the first phase, the numerical crisp model of the ML-MOFP problem has been developed at a confidence level without changing the fuzzy gist of the problem. Then, the linear model for the ML-MOFP problem is formulated. In the second phase, the interactive approach simplifies the linear multi-level multi-objective model by converting it into separate multi-objective programming problems. Also, each separate multi-objective programming problem of the linear model is solved by the ϵ-constraint method and the concept of satisfactoriness. Finally, illustrative examples and comparisons with the previous approaches are utilized to evince the feasibility of the proposed approach.

1. Introduction

Hierarchical decision structures are prevalent in government systems, competitive economic organizations, supply chains, agriculture, biofuel production, and so on (Baky, 2010). The area of multi-level mathematical programming (MLMP) provides the art and science of making such decisions. Several mathematical models for such problems have been exhibited (Abo-sinna and Baky, 2007; Baky, 2010; Osman et al., 2003; Pramanik and Roy, 2007). The fundamental idea of MLMP methodology is that the first-level decision maker (FLDM) decides his/her objectives and/or choices, hence asks each inferior level of the association for their solutions, which obtained individually. The lower level decision makers' choices are then presented and altered by the FLDM in light of the general advantage of the association (Abo-sinna and Baky, 2007; Baky, 2010).

As of late, MLMP has been deeply deliberated and several methods have been exhibited for solving such problems (Abo-sinna and Baky, 2007; Baky, 2010; Chen and Chen, 2015; Lachhwani 2015; Osman et al., 2003). An interactive algorithm for bi-level decision-making, problem has been proposed by Shi and Xia (1997). Interactive fuzzy programming has been extended by Sakawa et al. (2000a,b) to thoroughly consider in MLMP problems with fuzzy parameters. The balance space approach was extended to solve MLMP problems by Abo-Sinna and Baky (2007). Baky (2009) presented fuzzy goal programming (FGP) methodology to tackle decentralized bi-level programming problems (BL-PP). A further extension of the FGP approach for BL-PP with fuzzy demands was considered by Baky et al. (2014). Chen and Chen (2015) utilized a fuzzy variable for relative satisfactions among leader- and –follower to solve the decentralized BL-PP. Arora and Gupta (2009) exhibited interactive FGP methodology for BL-PP with the merits of dynamic programming.

Recently, Lachhwani (2015) proposed FGP approach introduced by Baky (2010) and Baky et al., 2014 with some modifications for ML-MOFP problems. An interactive FGP approach using Jacobian for decentralized bi-level multi-objective fractional programming has been displayed by Toksari and Bilim (2015). Osman et al. (2003) proposed interactive methodology for tri-level multi-objective programming problems. An interactive algorithm for a special class of two-level integer multi-objective fractional programming problem was studied by Emam (2013). Sakawa et al. (2000a,b) proposed interactive fuzzy programming for two-level linear fractional programming problems with fuzzy parameters. Also, Helmy et al. (2015) formulated a stochastic ML-MOFP problem in which the denominators at the same level must be identical. Osman et al. (2017) presented stochastic fuzzy ML-MOFP problem through FGP approach. Parametric ML-MOFP problem with

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fuzziness in the constraints presented by Osman et al. (2016). The proposed interactive mechanism for solving multi-level multi-objective decision-making problems simplifies these problems by changing them into isolated multi-objective decision-making problems at the different levels. In this way, the difficulty related to non-convex mathematical programming to get an optimal solution was avoided. Likewise, the algorithm raised the satisfactoriness concept as only for the FLDM predilection (Osman et al., 2003; Shi and Xia, 1997).

Fractional programming originates from the truth that programming paradigm could better fit the genuine problems if the optimization of a proportion between the physical and/or economic quantities is considered (Emam, 2013). In the course of recent decades, such problem has been one of the powerful planning tools. It is routinely used in engineering applications, business and different disciplines (Ahlatcioglu and Tiryaki, 2007; Lachhwani, 2015; Pal et al., 2003). Such type of problems in large hierarchical organizations of complex and conflicting multi-objectives formulate ML-MOFP problems.

During the formulation process of multi-objective decision-making problems, the coefficients of the objectives and the constraints are usually specified by specialists. In most genuine circumstances, the potential values of these coefficients are vague or ambiguously known to specialists. accordingly, it would be more convenient, for these coefficients to be represented as fuzzy numerical data (Baky et al., 2014; Sakawa et al., 2000a,b; Sakawa, 1993). The subsequent mathematical programming problem, including fuzzy coefficients would be seen as a more factual than the traditional one. From this perspective, the coefficients exist in the objective functions and the constraints of the ML-MOFP problem are thought to be described by fuzzy numbers.

The current research presents an interactive approach for solving ML-MOFP problems with fuzzy parameters. unlike the previous studies, in which the fractional objective functions at the same level must have identical denominators, in this study, we present the ML-MOFP problems with fuzzy parameters in the general formula. These parameters are expressed as a fuzzy number to represent the uncertainty in decision-making situation. This article also employs the α-cut (α-level) approach to formulate the crisp ML-MOFP problem. Then, we make further extension of the article presented by Chakraborty and Gupta, (2002) to obtain the linear model of the ML-MOFP problem. Moreover, we make use of the interactive approach to simplify the obtained linear model of the ML-MOFP problem by converting it into separate multi-objective decision-making problems. In addition to that, each separate multi-objective decision-making, problem of the linear model is solved by the ϵ-constraint method and the concept of satisfactoriness. Finally, An algorithm to clarify the developed interactive approach for solving the ML-MOFP problems with fuzzy parameters is introduced.

The remainder of this paper is marshaled as follows. Section 2 describes problem formulation of ML-MOFP problem with fuzzy parameters. It's equivalent crisp model and solution concepts are also established in Section 3. Section 4 develops the linear model of the α-(ML-MOFP) problem. The interactive models for solving α-(ML-MOFP) Problem are introduced in Section 5. An interactive algorithm for ML-MOFP Problem with Fuzzy parameters is proposed in Section 6. Numerical examples and comparisons with the existing approaches are provided in Section 7. This paper ends with some concluding remarks in Section 8.

2. Problem Formulation

Consider the hierarchical system be composed of a p-level decision maker. A decision maker is situated on each level, and a vector of fractional objective functions with fuzzy parameters needs to be optimized. Let the decision maker at the ith-level denoted by DM_i controls over the decision variable $\boldsymbol{x_i} = (x_{i1}, x_{i2}, \dots, x_{in_i}) \in R^{n_i}$, $i = 1, 2, \dots, p$. where $\boldsymbol{x} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_p}) \in R^n$ and $n = \sum_{i=1}^{p} n_i$ and furthermore it is assume that

$$F_i(\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_p}) \equiv F_i(\boldsymbol{x}): \; R^{n_1} \times R^{n_2} \times \dots \times R^{n_p} \to R^{k_i}, \quad i = 1, 2, \dots, p, \tag{1}$$

are the vector of fractional objective functions for $\text{DM}_i, i = 1, 2, \dots, p$. Mathematically, ML-MOFP problem with fuzzy parameters follows as (Baky et al., 2014; Osman et al., 2017; Pramanik and Roy, 2007):

[**1st *Level***]

$$\underbrace{max}_{\boldsymbol{x_1}} \tilde{F}_1(\boldsymbol{x}) = \underbrace{max}_{\boldsymbol{x_1}} (\tilde{f}_{11}(\boldsymbol{x}), \tilde{f}_{12}(\boldsymbol{x}), \dots, \tilde{f}_{1k_1}(\boldsymbol{x})), \tag{2}$$

where $\boldsymbol{x_2}, \boldsymbol{x_3}, \dots, \boldsymbol{x_p}$ *solves*

[**2nd *Level***]

$$\underbrace{max}_{\boldsymbol{x_2}} \tilde{F}_2(\boldsymbol{x}) = \underbrace{max}_{\boldsymbol{x_2}} \left(\tilde{f}_{21}(\boldsymbol{x}), \tilde{f}_{22}(\boldsymbol{x}), \dots, \tilde{f}_{2k_2}(\boldsymbol{x})\right), \tag{3}$$

$\vdots$

where $\boldsymbol{x_p}$ *solves*

[**pth *Level***]

$$\underbrace{max}_{\boldsymbol{x_p}} \tilde{F}_p(\boldsymbol{x}) = \underbrace{max}_{\boldsymbol{x_p}} (\tilde{F}_{p1}(\boldsymbol{x}), \tilde{F}_{p2}(\boldsymbol{x}), \dots, \tilde{F}_{pk_p}(\boldsymbol{x})), \tag{4}$$

subject to

$$\boldsymbol{x} \in \tilde{G} = \left\{ \boldsymbol{x} \in R^n \,\middle|\, \tilde{A}_1 \boldsymbol{x_1} + \tilde{A}_2 \boldsymbol{x_2} + \dots + \tilde{A}_t \boldsymbol{x_t} \begin{pmatrix} \leqslant \\ = \\ \geqslant \end{pmatrix} \tilde{\boldsymbol{b}}, \, \boldsymbol{x} \geqslant 0, \, \tilde{\boldsymbol{b}} \in R^m \right\}, \tag{5}$$

where

$$\tilde{f}_{ij}(\boldsymbol{x}) = \frac{\tilde{N}_{ij}(\boldsymbol{x})}{\tilde{D}_{ij}(\boldsymbol{x})} = \frac{\tilde{\boldsymbol{c}}_1^{ij} \boldsymbol{x_1} + \tilde{\boldsymbol{c}}_2^{ij} \boldsymbol{x_2} + \dots + \tilde{\boldsymbol{c}}_p^{ij} \boldsymbol{x_p} + \tilde{\alpha}^{ij}}{\tilde{d}_1^{ij} \boldsymbol{x_1} + \tilde{d}_2^{ij} \boldsymbol{x_2} + \dots + \tilde{\boldsymbol{d}}_p^{ij} \boldsymbol{x_p} + \tilde{\beta}^{ij}},$$
$$i = 1, 2, \dots, p, \quad j = 1, 2, \dots, k_i, \tag{6}$$

Also, all the parameters in the vectors $\tilde{\boldsymbol{c}}_l^{ij}, \tilde{\boldsymbol{d}}_l^{ij}, l = 1, 2, \dots, p$ and $\tilde{\boldsymbol{b}}$ are expressed as fuzzy numbers described by any form of membership functions, such as triangular or trapezoidal, depending on the DM's predilection. $\tilde{\boldsymbol{A}}_i$ represent the fuzzy coefficient matrices of size $m \times n_i, i = 1, 2, \dots, p$. It is usual to suppose that $\tilde{N}_{ij}(\boldsymbol{x}) \geqslant 0$,for some value of $\boldsymbol{x}$ $\tilde{D}_{ij}(\boldsymbol{x}) > 0, \forall x \in \tilde{G}$, also $\tilde{\alpha}^{ij}$ and $\tilde{\beta}^{ij}$ are fuzzy scalars and $\tilde{G}$ represents the convex constraints feasible choice set in the fuzzy environment.

3. Formulation of crisp model and solution concept

Based on ML-MOFP model (2)–(5), the coefficients in the objective functions and the set of constraints are represented by fuzzy numbers. Let $\mu_{\tilde{\boldsymbol{c}}_l^{ij}}, \mu_{\tilde{\boldsymbol{d}}_l^{ij}}, \mu_{\tilde{\alpha}^{ij}}, \mu_{\tilde{\beta}^{ij}}, \mu_{\tilde{\boldsymbol{b}}}$ and $\mu_{\tilde{\boldsymbol{A}}_i}$ be the membership functions which represent the fuzzy numbers in the corresponding vectors $\tilde{\boldsymbol{c}}_l^{ij}, \tilde{\boldsymbol{d}}_l^{ij}, \tilde{\alpha}^{ij}, \tilde{\beta}^{ij}$ and $\tilde{\boldsymbol{b}}$ and the fuzzy coefficient matrices $\tilde{\boldsymbol{A}}_i$, respectively. Generally, a fuzzy set can be completely represented by its α-cuts (Baky et al., 2014; Chen and Chen, 2015; Sakawa, 1993), $\alpha \in [0, 1]$, which are known as the numerical intervals in which the membership levels of the elements belong to the fuzzy set exceeds or equal to the α-level. The α-cuts of $\tilde{\boldsymbol{c}}_l^{ij}, \tilde{\boldsymbol{d}}_l^{ij}, \tilde{\alpha}^{ij}, \tilde{\beta}^{ij}, \tilde{\boldsymbol{b}}$ and $\tilde{\boldsymbol{A}}_i$ are defined as (Baky et al., 2014; Chen and Chen, 2015; Sakawa, 1993):