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Parameter identification for nonlinear damping coefficient from large-amplitude ship roll motion using wavelets

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ABSTRACT

In this paper, we have introduced an efficient Legendre wavelet spectral method (LWSM) to ship roll motion model for investigating the nonlinear damping coefficients. The accuracy of the proposed method is assessed by rolling with an initial amplitude using nonlinear type of equations. To the best of our knowledge until now there is no rigorous wavelet solution has been reported for the above model. Convergence analysis of the proposed method is discussed. The operational matrix of derivative is utilized to convert a ship roll motion equation into a set of algebraic equations. Accurate coefficients can be easily obtained for large rolling angles up to the amplitude at the maximum value of the restoring moment using wavelets. Some numerical examples are given to show the validity and applicability of the proposed wavelet method. Moreover the use of Legendre wavelets is found to be simple,flexible, accurate, efficient and small computation cost.

1. Introduction

Ship roll motion has been attracting considerable attention over the past ten years because of it being the most critical motion loading to a capsize, out of six motions of a ship (Taylan, 1999, 2000).

In recent years, the mathematical modeling of ship motion at sea has continued to attract among researchers (Cotton and Spyrou, 2001; Haddara and Wu, 1993; Taylan, 1999). Chan et al. (1995) had developed the Runge–Kutta (R–K) algorithm for determining the nonlinear damping coefficients from large-amplitude ship rolling motions. In order to compute the coefficients in the damping terms, various numerical methods have been developed. Spouge (1988) established the perturbation method to determine the damping coefficients. Chang (2008) had applied the simulation method for determining the damping coefficients. Non-linear formulation of the ship roll motion can be investigated in the literature (Cohen, 1999; Fossen, 2011; Mathiesen and Price, 1984; Roberts, 1985). Various nonlinear models have been studied by many researchers (Surendran and Venkata Ramana Reddy, 2002; Surendran and Venkata Ramana Reddy, 2003). Linearity of the ship motion is often violated by the non-linear features of damping and restoring. The estimation of the roll damping coefficients continues to raise a lot of new problems. Froude (1872) was one of the important pioneers of obtaining ship roll damping coefficients from experimental data using a single-degree-of-freedom equation. Perez and Blanke (2012) reviewed the assessment of performance and the applicability of different mathematical models and control methods. Yang et al. (2012) established the CFD simulation for roll damping motions. Based on CFD, warship model's roll damping motions are simulated, and a method was proposed to vessel's roll damping coefficient. Parametric identification of the roll motion equation initiated as the only way for estimating the roll damping parameters (Froude, 1872). The parametric identification problem was extended to obtain estimates for the restoring moment from measurements of roll motion excited by random waves (Haddara and Wu, 1993). Haddara and Wishahy (2002) used the regression techniques for estimating the damping and the nonlinear parameters in the restoring moment. Haddara and Bennett (1989) had investigated the angle dependence of the roll damping moment. Chakrabarti (2001) examined the roll damping components and their empirical contributions.

Ship roll motion parameters have been studied using small scale models in a towing tank environment. Various prediction models of roll motion containing non-linear terms in damping and restoring have been addressed by many researchers (Francescutto, 2000; Taylan, 1999; Taylan, 2000). Several approximation methods have been implemented to handle such non-linear equations in both the frequency and time domains.

The rolling angle θ is assumed to satisfy a non-linear equation of motion of the form

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$$I\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + K(\theta) = 0 \quad (1.1)$$

where I is the total inertia, C and K are arbitrary functions of their arguments and a dot indicates a derivative with respect to time (Chan et al., 1995). The roll inertia I, which includes the added inertia of water, will usually be taken as a constant. Both the inertia and the restoring moment $K(\theta)$ can be calculated from hydrostatic considerations. An odd polynomial is usually assumed as the form for $K(\theta)$, whereas the form for the damping term $C\dot{\theta}$ is somewhat uncertain. Commonly adopted forms include linear-plus-quadratic and linear-plus-cubic expressions, as presented by Dalzell (1978).

Recently, wavelet transform is a new branch of mathematics is widely used in signal analysis, image compression and numerical analysis etc. It has found their way into many different fields in science and engineering (Cohen, 1999; Hariharan and Kannan, 2014; Kuramoto, 1984; Mallat, 1999) Wavelets theory possesses some useful properties, such as compact support, orthogonality, dyadic, orthonormality and multi-resolution analysis (MRA). In wavelet based numerical methods, there are two important ways of improving the approximation of the solutions; increasing the order of the wavelet family and increasing the resolution level of the wavelet. Among the wavelet families the Haar, Legendre, Bernoulli and Chebyshev wavelets deserve much attention. The basic idea of Legendre wavelet method (LWM) is to convert the nonlinear differential equations to a system of algebraic equations by the operational matrices of integral or derivative.

Wavelet transform, as very well-localized functions, are considerably useful tool for solving differential equations and providing excellent solutions (Hariharan, 2014a) Also,the wavelet based technique gives very fast algorithms when compared with the other algorithms ordinarily used (Farge, 1992; Hariharan, 2014a). The pioneer work in lumped and distributed parameter via Haar wavelets was led by Chen and Hsiao (1997), who first derived the operational matrices for the integrals. The Legendre wavelets is constructed from Legendre polynomials and form a basis for $\mathscr{L}^2(\mathbb{R})$ over $[0,1]$. Parsian (2005) used the Legendre wavelet method for variational problems. Razzaghi and Yousefl (2000) had introduced the Legendre wavelet method for solving variational problems. Hariharan and his coworkers (2014a,b) had introduced the wavelet based methods for solving differential equations arising in engineering. Hariharan (2014b) introduced the coupled Legendre wavelet method for Newell–Whitehead and Allen–Cahn equations. Fukang et al. (2015) developed the spectral methods using Legendre wavelets for nonlinear Klein–Gordon equations. Li and Guo (1997) had applied the Legendre spectral method for solving the nonlinear Klein–Gordon equations. Yousefi (2006) established the Legendre wavelet method for solving Lane–Emden type differential equations. Maleknejad and Sohrabi (2007) had used the Legendre wavelet method for solving Fredholm integral equations of the first kind. Mohammadi and Hosseini (2011) had applied the Legendre wavelet based operational matrix method for singular ODEs. Recently, Hariharan and Kannan (2014) reviewed the wavelet methods for the solution of Reaction–Diffusion problems arising in science and engineering. They discussed a lot engineering applications of wavelet methods in solving differential equations. Some excellent references are therein. Karimi Dizicheh et al. (2013) established the Legendre wavelet spectral collocation method for solving oscillatory initial value problem. Hariharan and Kannan (2010, 2013) and Hariharan and Rajaraman (2013) presented the overview of Haar wavelet method for solving differential and integral equations, some non-linear parabolic equations and estimating depth profile of soil temperature. Burger and Ruiz-Baier (2009) discuss the multi-resolution simulation of reaction–diffusion systems with strong degeneracy. The Yin et al. (2012) and Yin Yu et al. (2006) established coupled method of Laplace transform and Legendre wavelets for Lane–Emden-type differential equations. Aziz et al. (2013) used wavelets collocation methods for the numerical solution of elliptic BV problems. Hariharan and Rajaraman (2013) had discussed the coupled wavelet-based method applied to the nonlinear reaction–diffusion equation arising in mathematical chemistry. Yin Yu et al. (2006) implemented the wavelet method to analyze nonlinear ship rolling and heave-roll coupling. In order to demonstrate the accuracy and efficiency of the proposed wavelet approach, sample nonlinear equations of motion are assumed to be given. The proposed wavelet method is used to analyse the envelope of the simulated responses. Parametric studies are then performed using initial roll angles of increasing amplitude.

This paper is summarized as follows. In Section 2, we discuss the basic properties of Legendre wavelets method (LWM) and Legendre polynomials. Multi-resolution analysis (MRA) is given in Section 3. We apply the Legendre wavelets on the spectral method for solving non-linear ship roll motion problems in Section 4. Concluding remarks are given in Section 5.

2. Wavelets and Legendre wavelets

Wavelets constitute a family of function constructed from dilation and translation of a single function called the mother wavelet (Goswami and Chan, 1999; Kheiri and Ghafouri., 2014; Mallat, 1999). When the dilation parameter a and the translation parameter b vary continuously we have the following family of continuous wavelets as (Gu and Jiang, 1996)

$$\psi_{a,b}(t) = |a|^{-\frac{1}{2}}\psi\left(\frac{t-b}{a}\right), a, b \in \mathbb{R}, a \neq 0.$$

If we restrict the parameters a and b to discrete values as $a = a_0^{-k}, b = nb_0a_0^{-k}, a_0 > 1, b_0 > 0$and n, and k positive integers, we have the following family of discrete wavelets:

$$\psi_{k,n}(t) = |a|^{\frac{k}{2}}\psi(a_0^k t - nb_0),$$

where $\psi_{k,n}(t)$ forms a wavelet basis for $\mathscr{L}^2(\mathbb{R})$. In particular, when $a_0 = 2$ and $b_0 = 1$, then $\psi_{k,n}(t)$ forms an orthonormal basis (Gu and Jiang, 1996)

Legendre wavelets $\psi_{k,n}(t) = \psi(k, \hat{n}, m, t)$ have four arguments; $\hat{n} = 2n-1, n = 1,2,3,\ldots,2^{k-1}, k$ can assume any positive integer, m is the order for Legendre polynomials and t is the normalized time. They are defined (Razzaghi and Yousefl, 2000) on the interval $[0,1]$ as

$$\psi_{n,m}(t) = \begin{cases} \sqrt{m+\frac{1}{2}}2^{\frac{k}{2}}P_m(2^k t - \hat{n}) & \text{for } \frac{\hat{n}-1}{2^k} \leqslant t \leqslant \frac{\hat{n}+1}{2^k} \\ 0 & \text{for otherwise} \end{cases} \quad (2.1)$$

where $m = 0,1,\ldots,M-1, n = 1,2,\ldots,2^{k-1}$. In Eq. (2.1), the coefficient $\sqrt{m+\frac{1}{2}}$ is for orthonormality, the dilation parameter is $a = 2^{-k}$ and translation parameter is $b = \hat{n}2^{-k}$. Here, $P_m(t)$ are the well-known Legendre polynomials of order m which are orthogonal with respect to the weight function $w(t) = 1$ on the interval $[-1,1]$, and satisfy the following recursive formula:

$$P_0(t) = 1, P_1(t) = t,$$

$$P_{m+1}(t) = \left(\frac{2m+1}{m+1}\right)tP_m(t) - \left(\frac{m}{m+1}\right)P_{m-1}(t), \quad m = 1,2,3,\ldots$$

2.1. Function approximation

A function $f(t)$ defined over $[0,1)$ may be expanded (Hariharan and Rajaraman, 2013) as