BENI-SUEF UNIVERSITY JOURNAL OF BASIC AND APPLIED SCIENCES 5 (2016) 306–309

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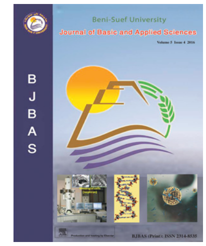

Full Length Article

Approximation of analytic functions in adaptive environment

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ARTICLE INFO

Article history:
Received 12 March 2016
Accepted 18 October 2016
Available online 2 November 2016

Keywords:
Analytic functions
Mixed quadrature rule
Degree of precision
Error bound
Adaptive quadrature

ABSTRACT

In this paper, a mixed quadrature rule of degree of precision seven is formed for analytic functions by taking two constituent rules each of degree of precision five. Here the integral of analytic function is converted to real definite integrals with the help of double transformations. Then the mixed quadrature rule is tested in adaptive environment and it is obviously superior to that of Gauss–Legendre three-point rule.

1. Introduction

The integral of the type

$$I(f)=\int_L f(z)dz \tag{1.1}$$

where L is the directed line segment from the point z_0-h to z_0+h in the complex plane and $f(z)$ is analytic in certain domain Ω containing the line segment L. Lether using the transformation $z=z_0+th$ where $t\in[-1,1]$ transforms the integral Eq. (1.1) to

$$I(f)=h\int_{-1}^{1} f(z_0+th)dt \tag{1.2}$$

Here we adopt another transformation $z=th$ in Eq. (1.2) which transforms analytic functions to real definite integrals of the form

$$I(f)=\int_a^b f(x)dx \tag{1.3}$$

Using the monomial transformation $2x=(b-a)t+(b+a)$ in Eq. (1.3), it transforms to

$$I(f)=\int_{-1}^{1} f(t)dt \tag{1.4}$$

For a real integrable function f on interval $[a,b]$ and a prescribed tolerance ε, it is desired to compute an approximation p to the integral Eq. (1.3) so that $|p-I|\le\varepsilon$. The basic principle

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http://dx.doi.org/10.1016/j.bjbas.2016.10.001

of adaptive quadrature scheme is discussed in the following manner.

If c is in between a and b, then

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

We design an algorithm for numerical computation of integrals in the adaptive quadrature rules. To prepare an algorithm for adaptive quadrature scheme in evaluating an integral Eq. (1.3) we use the mixed quadrature rule $R_{WGL3}(f)$ as I_1.

1.1. *Adaptive algorithm*

The inputs to the algorithm are a, b, n, f. The output is $p \approx \int_a^b f(x)dx$ with an error less than ε; n is the number of intervals chosen.

Step-1: An approximation I_1 to $I = \int_a^b f(x)dx$ is calculated.

Step-2: The interval $[a, b]$ is divided into two sub-intervals $[a, c]$ and $[c, b]$ where $c = \frac{a+b}{2}$. Then $I_2 \approx \int_a^c f(x)dx$ and $I_3 \approx \int_c^b f(x)dx$ are computed.

Step-3: $I_2 + I_3$ is compared with I_1 to estimate the error in $I_2 + I_3$. If $|estimated\ error| \le \frac{\varepsilon}{2}$(termination criterion) then $I_2 + I_3$ is accepted as an approximation to $I = \int_a^b f(x)dx$. Otherwise, the same procedure is applied to $[a, c]$ and $[c, b]$ allowing each piece a tolerance of $\frac{\varepsilon}{2}$. If the termination criterion is not satisfied on one or more of the sub-intervals then those sub-intervals must further be sub-divided and the entire process is repeated.

Keeping the facts in mind of so many researchers (Birkhoff and Young, 1950; Das and Pradhan, 1996; Jena and Dash, 2011; Mat et al., 1996; Tosic, 1978; Das and Pradhan, 1997; Senapati et al., 2011), here we formulate a mixed quadrature rule of degree of precision seven by taking linear convex combination of two constituent rules namely, Weddle's transformed rule and Gauss-Legendre three-point rule each of degree of precision five. The mixed rule so formed is numerically implemented on different integrals executing better results than Gauss-Legendre three-point rule in adaptive environment and also the error bound is determined.

2. Construction of mixed quadrature rule

Weddle's transformed rule $R_W(f)$ is

$$\begin{aligned} I(f) &\cong R_W(f) \\ &= \frac{h}{10}\left[f(z_0 - h) + f(z_0 + h) + f\left(z_0 - \frac{h}{3}\right) + f\left(z_0 + \frac{h}{3}\right) \right. \\ &\quad \left. + 5f\left(z_0 - \frac{2h}{3}\right) + 5f\left(z_0 + \frac{2h}{3}\right) + 6f(z_0) \right] \end{aligned} \tag{2.1}$$

Expanding each term of Eq. (2.1) using Taylor series about z_0

$$\begin{aligned} R_W(f) = 2h\Big[& f(z_0) + \frac{h^2}{3!}f''(z_0) + \frac{h^4}{5!}f^{iv}(z_0) \\ & + \frac{35h^6}{243\times 6!}f^{vi}(z_0) + \frac{1307h^8}{10935\times 8!}f^{viii}(z_0) + \ldots\Big] \end{aligned} \tag{2.2}$$

We can write Eq. (1.1) using Taylor series expansion about z_0

$$\begin{aligned} I(f) = 2h\Big[& f(z_0) + \frac{h^2}{3!}f''(z_0) + \frac{h^4}{5!}f^{iv}(z_0) + \frac{h^6}{7!}f^{vi}(z_0) \\ & + \frac{h^8}{9!}f^{viii}(z_0) + \ldots\Big] \end{aligned} \tag{2.3}$$

Now the error associated with Weddle's rule $R_W(f)$ is

$$\begin{aligned} E_W(f) &= I(f) - R_W(f) \\ &= -\frac{4h^7}{1701\times 6!}f^{vi}(z_0) - \frac{184h^9}{10935\times 8!}f^{viii}(z_0) \end{aligned} \tag{2.4}$$

Similarly, Gauss–Legendre three point rule $R_{GL3}(f)$ is

$$\begin{aligned} I(f) \cong R_{GL3}(f) = \frac{h}{9}\Big[& 5f\left(z_0 - \sqrt{\frac{3}{5}}h\right) + 8f(z_0) \\ & + 5f\left(z_0 + \sqrt{\frac{3}{5}}h\right)\Big] \end{aligned} \tag{2.5}$$

Expanding each term of Eq. (2.5) using Taylor series about z_0

$$\begin{aligned} R_{GL3}(f) = 2h\Big[& f(z_0) + \frac{h^2}{3!}f''(z_0) + \frac{h^4}{5!}f^{iv}(z_0) \\ & + \frac{3h^6}{25\times 6!}f^{vi}(z_0) + \frac{9h^8}{125\times 8!}f^{viii}(z_0) + \ldots\Big] \end{aligned} \tag{2.6}$$

The error associated with the Gauss–Legendre three-point rule $R_{GL3}(f)$ is

$$\begin{aligned} E_{GL3}(f) &= I(f) - R_{GL3}(f) \\ &= \frac{8h^7}{175\times 6!}f^{vi}(z_0) + \frac{88h^9}{1125\times 8!}f^{viii}(z_0) \end{aligned} \tag{2.7}$$

2.1. *Mixed quadrature rule* $R_{WGL3}(f)$ *of degree of precision seven*

Using Taylor series expansion of Eq. (2.1) and Eq. (2.5), referring to Jena and Dash (2015) and Milovanovic et al. (2015) we get,

$$I(f) = R_W(f) + E_W(f) \tag{2.8}$$

$$I(f) = R_{GL3}(f) + E_{GL3}(f) \tag{2.9}$$

Now multiplying $\left(\frac{2}{25}\right)$ in Eq. (2.8) and $\left(\frac{1}{243}\right)$ in Eq. (2.9) and adding,

$$I(f) = R_{WGL3}(f) + E_{WGL3}(f) \tag{2.10}$$