ARTICLE IN PRESS

BENI-SUEF UNIVERSITY JOURNAL OF BASIC AND APPLIED SCIENCES ■■ (2016) ■■–■■

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Full Length Article

Iterative method for approximate solution of fuzzy integro-differential equations

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ARTICLE INFO

Article history:
Received 4 July 2016
Received in revised form 23 October 2016
Accepted 4 November 2016
Available online

Keywords:
Nonlinear fuzzy integro-differential equations
Newton–Cotes methods
Parametric form of a fuzzy number

ABSTRACT

In this paper, we interpret a nonlinear fuzzy Fredholm integro-differential equations by using the strongly generalized differentiability concept. Based on the parametric form of a fuzzy number, a fuzzy integro-differential equation converts to two systems of integro-differential equations in the crisp case. Also, we use the parametric form of fuzzy number, and an iterative approach for obtaining approximate solution for a class of nonlinear fuzzy Fredholm integro-differential equation of the second kind is proposed. This paper presents a method based on Newton–Cotes methods with positive coefficient. Then we obtain approximate solution of the nonlinear fuzzy integro-differential equations by an iterative approach.

1. Introduction

The study of fuzzy integral equations (FIE) forms a suitable setting for mathematical modeling of real-world problems in which uncertainties or vagueness pervade. The topics of fuzzy integral equations which is a growing interest for some time, in particular in relation to fuzzy control, have been rapidly developed in recent years. The fuzzy mapping function was introduced by Chang and Zadeh (1972). Later, Dubois and Prade (1982) presented an elementary fuzzy calculus based on the extension principle also the concept of integration of fuzzy functions.

Since some of the fuzzy integral equations cannot be solved explicitly, it is often necessary to resort to numerical techniques which are appropriate combinations of numerical integration and interpolation. There are several numerical methods for solving fuzzy linear Fredholm integral equation (Abbasbandy et al., 2007; Babolian et al., 2005; Friedman et al., 1999) and fuzzy differential equation (Mosleh and Otadi, 2012a, 2012b). Hosseini Fadravi et al. (2014) considered solutions of fuzzy Fredholm integral equations using neural networks. Based on the parametric form of a fuzzy number, a Fredholm fuzzy integral equation converts to two systems of integral equations of the second kind in the crisp case. In their approach, they grow neural network as the approximate solution of the integral equations, for which the activation functions are log-sigmoid and linear functions. Recently Otadi and Mosleh (2016) introduced new generalized interval-valued fuzzy linear Fredholm integral equation concepts. The work of this paper

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http://dx.doi.org/10.1016/j.bjbas.2016.11.008

is an expansion of the research of real fuzzy linear Fredholm integral equations. In this paper interval-valued fuzzy neural network (IVFNN) can be trained with crisp and interval-valued fuzzy data. In this paper, a novel hybrid method based on IVFNN and Newton–Cotes methods with positive coefficient for the solution of interval-valued fuzzy linear Fredholm integral equations (IVFLFIEs) of the second kind is presented. Within this paper the fuzzy neural network model is used to obtain an estimate for the fuzzy parameters in a statistical sense. Also Mosleh and Otadi (2016) proved some results concerning the existence of solution for a class of nonlinear fuzzy Fredholm integro-differential equations. Also an iterative approach to obtain approximate solution for a class of nonlinear fuzzy Fredholm integro-differential equation of the second kind is proposed. But they considered the H-derivative of fuzzy numbers.

While finding an approximate solution for the nonlinear fuzzy integro-differential equations

$$X'(s) = y(s) + \int_a^b k(s, t, X(t))\,dt,$$

is difficult.

In this paper, using strongly generalized differentiability, we present a novel and very simple numerical method based on iterative methods for solving nonlinear fuzzy Fredholm integro-differential equations of the second kind. The strongly generalized differentiability was introduced in Bede and Gal (2004) and studied in Bede and Gal (2005). This concept allows us to resolve the above-mentioned shortcoming. Indeed, the strongly generalized derivative is defined for a larger class of fuzzy-number-valued functions than the Hukuhara derivative. Hence, we use this differentiability concept in the present paper.

2. Preliminaries

In this section the basic notations used in fuzzy operations are introduced. We start by defining the fuzzy number.

Definition 1. A fuzzy number is a fuzzy set $u:\mathbb{R}^1 \to I = [0,1]$ such that

i. u is upper semi-continuous;
ii. $u(x) = 0$ outside some interval $[a,d]$;
iii. there are real numbers b and c, $a \le b \le c \le d$, for which
   1. $u(x)$ is monotonically increasing on $[a,b]$,
   2. $u(x)$ is monotonically decreasing on $[c,d]$,
   3. $u(x) = 1, b \le x \le c$.

The set of all the fuzzy numbers (as given in Definition 1) is denoted by E (Klir et al., 1997).

An alternative definition which yields the same E^1 is given by Kaleva (1987).

Definition 2. A fuzzy number u is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r)$ and $\overline{u}(r)$, $0 \le r \le 1$, which satisfy the following requirements:

i. $\underline{u}(r)$ is a bounded monotonically increasing, left continuous function on (0,1] and right continuous at 0;
ii. $\overline{u}(r)$ is a bounded monotonically decreasing, left continuous function on (0,1] and right continuous at 0;
iii. $\underline{u}(r) \le \overline{u}(r), 0 \le r \le 1$.

A crisp number r is simply represented by $\underline{u}(\alpha) = \overline{u}(\alpha) = r, 0 \le \alpha \le 1$. The set of all the fuzzy numbers is denoted by E^1. This fuzzy number space as shown in Congxin and Ming (1991) can be embedded into the Banach space $B = \overline{C}[0,1] \times \overline{C}[0,1]$.

For arbitrary $u = (\underline{u}(r), \overline{u}(r)), v = (\underline{v}(r), \overline{v}(r))$ and $k \in \mathbb{R}$ we define addition and multiplication by k as

$$\underline{(u+v)}(r) = (\underline{u}(r) + \underline{v}(r)),$$
$$\overline{(u+v)}(r) = (\overline{u}(r) + \overline{v}(r)),$$
$$\underline{ku}(r) = k\underline{u}(r), \overline{ku}(r) = k\overline{u}(r), \text{ if } k \ge 0,$$
$$\underline{ku}(r) = k\overline{u}(r), \overline{ku}(r) = k\underline{u}(r), \text{ if } k < 0.$$

Definition 3. For arbitrary fuzzy numbers u,v, we use the distance (Goetschel and Vaxman, 1986)

$$D(u, v) = sup_{0\le r\le 1} max\{|\overline{u}(r) - \overline{v}(r)|, |\underline{u}(r) - \underline{v}(r)|\}$$

and it is shown that (E,D) is a complete metric space (Puri and Ralescu, 1986).

Definition 4. Let $f:[a, b] \to E$, for each partition $P = \{t_0, t_1, \dots, t_n\}$ of $[a,b]$ and for arbitrary $\xi_i \in [t_{i-1}, t_i], 1 \le i \le n$ suppose

$$R_p = \sum_{i=1}^{n} f(\xi_i)(t_i - t_{i-1}),$$
$$\Delta := max\{|t_i - t_{i-1}|, i = 1, 2, \dots, n\}.$$

The definite integral of $f(t)$ over $[a,b]$ is

$$\int_a^b f(t)\,dt = lim_{\Delta\to 0} R_p$$

provided that this limit exists in the metric D (Friedman et al., 1999).

If the fuzzy function $f(t)$ is continuous in the metric D, its definite integral exists (Goetschel and Vaxman, 1986) and also,

$$\underline{\left(\int_a^b f(t;r)\,dt\right)} = \int_a^b \underline{f}(t;r)\,dt,$$
$$\overline{\left(\int_a^b f(t;r)\,dt\right)} = \int_a^b \overline{f}(t;r)\,dt.$$

Definition 5. Let $u,v \in E$. If there exists $w \in E$, such that $u = v + w$, then w is called the H-difference of u,v and it is denoted $u \ominus v$.

In this paper the sign $\ominus$ always stands for the H-difference, and let us remark that $u \ominus v \ne u + (-1)v$. Usually we denote $u + (-1)v$ by $u - v$, while $u \ominus v$ stands for the H-difference. In what follows, we fix $T = (a,b)$, for $a,b \in \mathbb{R}$.

Definition 6. Let $F:T \to E$ be a fuzzy function. We say F is differentiable at $t_0 \in T$ if there exists an element $F'(t_0) \in E$ such that the limits

Please cite this article in press as: Mahmood Otadi, Maryam Mosleh, Iterative method for approximate solution of fuzzy integro-differential equations, beni-suef university journal of basic and applied sciences (2016), doi: 10.1016/j.bjbas.2016.11.008