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Extended modified cubic B-spline algorithm for nonlinear Burgers' equation

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ABSTRACT

In this paper, an extended modified cubic B-Spline differential quadrature method is proposed to approximate the solution of the nonlinear Burgers' equation. The proposed method is used in space and a five-stage and four order strong stability-preserving time-stepping Runge–Kutta (SSP-RK54) method is used in time. The accuracy and efficiency of the method is illustrated by considering four numerical problems. The numerical results of the method are compared with some existing methods and it was found that the proposed numerical method produces acceptable results and even more accurate results in comparison with some existing methods. The stability analysis of the scheme is also carried out and was found to be unconditionally stable.

1. Introduction

Consider one dimensional nonlinear Burgers' equation

$$\frac{\partial u}{\partial t}+\alpha u\frac{\partial u}{\partial x}-\upsilon\frac{\partial^2 u}{\partial x^2}=0;\quad a_1\le x\le a_2,\quad t\ge 0, \tag{1.1}$$

with initial conditions:

$$u(x,0)=\phi(x);\quad a_1\le x\le a_2, \tag{1.2}$$

and Dirichlet boundary conditions:

$$u(a_1,t)=0=u(a_2,t);\ t\ge 0, \tag{1.3}$$

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and two dimensional nonlinear coupled Burgers' equations

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \quad (1.4)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \quad (1.5)$$

with initial conditions:

$$u(x,y,0)=\varphi_1(x,y) \text{ and } v(x,y,0)=\varphi_2(x,y), \quad (x,y)\in\Omega, \quad (1.6)$$

and Dirichlet boundary conditions:

$$u(x,y,t)=\psi_1(x,y,t) \text{ and } v(x,y,t)=\psi_2(x,y,t), \quad (x,y)\in\partial\Omega, t>0, \quad (1.7)$$

where $\Omega=\{(x,y): a_1\le x\le a_2, \quad b_1\le x\le b_2\}$ is the computational square domain and $\partial\Omega$ is its boundary, $u(x,t)$ is the velocity component in one dimension, $u(x,y,t)$ and $v(x,y,t)$ are the velocity components in two dimension; ϕ, φ_1, φ_2, ψ_1 and ψ_2 are known functions; $\frac{\partial u}{\partial t}$ is unsteady term, $u\frac{\partial u}{\partial x}$ is the nonlinear convection term, $\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$ is the diffusion term and $\upsilon>0$ is the coefficient of viscosity and α is the some positive constant.

The Burgers' equation is an easiest model for explaining various physical flows problems, such as hydrodynamic turbulence, sound and shock wave theory, vorticity transportation, dispersion in porous media, modeling of turbulent fluid, etc, Burgers (1939), Cole (1951), Esipov (1995). First of all, Bateman (1915) introduced this type of equations and later a steady-state solution was proposed by Burgers (1939).

The Burgers' equation has been solved by various analytical and numerical schemes such as Hofe Cole transformation (Cole, 1951; Fletcher, 1983),finite element method (Aksan, 2006; Cecchi et al., 1996; Dogan and Galerkin, 2004; Ozis et al., 2003), finite difference method (Hassanien et al., 2005), explicit finite difference method (Kutluay et al., 1999), implicit finite difference method (Kadalbajoo et al., 2005), compact finite difference method (Liao, 2008), implicit logarithmic finite difference method (Srivastava et al., 2013), least-squares quadratic B-spline finite element method (Kutluay et al., 2004), quadratic B-spline finite elements (Ozis et al., 2005), B-Spline collocation method (Dag et al., 2005), quartic B-spline collocation method (Saka and Dag, 2007), reproducing kernel function method (Xie et al., 2008), cubic B-spline quasi-interpolation (Jiang and Wang, 2010), sinc differential quadrature method (Korkmaz and Dag, 2011), Fourier expansion-based differential quadrature method (Shu and Chew, 1997), quartic B-spline differential quadrature method (Korkmaz et al., 2011), modified cubic B-splines collocation method (Mittal and Jain, 2012), cubic B-spline differential quadrature methods (Korkmaz and Dag, 2013), and modified cubic B-spline differential quadrature methods (Arora and Singh, 2013; Shukla et al., 2014).

Doha et al. (2014) presented a numerical solution of the nonlinear coupled viscous Burgers' equation based on spectral methods. They employed a Jacobi–Gauss–Lobatto collocation scheme in combination with the implicit Runge–Kutta–Nyström scheme. Bhrawy et al. (2015) proposed new space-time spectral algorithm based on spectral shifted Legendre collocation method in combination with the shifted Legendre operational matrix of fractional derivatives to approximate the solution for the space-time fractional Burgers' equation. Bhrawy and Zaky (2016) proposed a new approach based on the shifted Chebyshev polynomials to approximate the solution of functional variable-order fractional differential equations. Esen and Tasbozan (2016) used collocation method based on cubic B-spline basis functions for the time fractional Burgers' equation.

Bellman et al. (1972) were the first to introduce an efficient technique named "differential quadrature method (DQM)" for the solution of PDEs. It was further improved by Quan and Chang (1989) to approximate the weighting coefficients. In DQM, several kinds of test functions have been used to compute the weighting coefficients viz. B-spline function, cubic B-spline functions, sinc function, Lagrange interpolation polynomials, Legendre polynomials, quartic B-spline functions, modified cubic B-spline functions, etc. B-splines are a set of certain functions that can be used to build piece-wise polynomial by computing the suitable linear combination. These basis functions have more influence in comparison to other basis functions due to its smoothness and capability to handle local phenomena. Recently, Dag et al. (2013) proposed an extended cubic B-spline algorithm for numerical solution of a modified regularized long wave equation.

The main objective of this study is to present a new method, namely, an extended modified cubic-B-spline differential quadrature method (EMCB-DQM) for the numerical computation of the Burgers' equation. In this method, the extended modified cubic-B-spline basis functions are used in DQM to determine the weighting coefficients which transform the Burgers' equation into a system of first order ordinary differential equations. The resulting system of equations is solved by employing a five-stage and four order strong stability-preserving time-stepping Runge–Kutta method. The efficacy and adaptability of the method is confirmed by taking a four test problem in one and two dimensions. The rest of the paper is prepared as: in Section 2, the EMCB-DQM is introduced; in Section 3, implementation procedure to the Burgers' equation is illustrated; in Section 4, stability analysis is discussed; in Section 5, four test problems are considered in order to establish the applicability and accuracy of the proposed method, while Section 6 concludes our study.

2. Extended modified cubic B-spline differential quadrature method

For one dimensional Burgers' equation, let us assume that N grid points/knots $a_1=x_1<x_2,\dots<x_N=a_2$ are uniformly distributed with step size $\Delta x=x_{i+1}-x_i$ along x direction. The rth order spatial partial derivatives of the unknown $u(x,t)$ with respect to x, approximated at x_i, $i=1,2,\dots,N$, are defined as

$$\frac{\partial^r u(x_i,t)}{\partial x^r}=\sum_{k=1}^{N} a_{ij}^{(r)}u(x_j,t), \qquad i=1,2,\dots,N \quad (2.1)$$

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