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Band gap behaviour of optimal one-dimensional composite structures with an additive manufactured stiffener

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ABSTRACT

In this work, the banded behaviour of composite one-dimensional structures with an additive manufactured stiffener is examined. A finite element method is used to calculate the stiffness, mass and damping matrices, and periodic structure theory is used to obtain the wave propagation of one-dimensional structures. A multi-disciplinary design optimisation scheme is developed to achieve optimal banded behaviour and structural characteristics of the structures under investigation. Having acquired the optimal solution of the case study, a representative specimen is manufactured using a carbon fibre cured plate and additive manufactured nylon-based material structure. Experimental measurements of the dynamic performance of the hybrid composite structure are conducted using a laser vibrometer and electrodynamic shaker setup to validate the finite element model.

1. Introduction

Noise and vibration transmission within payload and passenger compartments is a major issue for modern transport vehicles. To ensure the quality of their products, manufacturers in the transport industry are simultaneously trying to optimise the mechanical and the vibroacoustic performance of structural assemblies. It has been demonstrated that judiciously designed periodic structures can induce vibration attenuation and stop-band behaviour in specific frequency ranges (so-called *band gaps* or *stop bands*).

Floquet [1] was the first to publish on periodic structures, in which the one-dimensional (1D) Mathieu's equations were studied to predict band gap behaviour. Floquet's work was followed by that of Rayleigh [2], who developed a similar form to Floquet's theorem. During the twentieth century, Mead [3,4], Mace et al. [5] and Langley and Cotoni et al. [6,7] produced mathematical tools based on Brillouin's periodic structure theory (PST) [8]. Using these methods, researchers have the ability to predict the vibroacoustic and dynamic performance of several applications in relatively short times. Application examples are presented with composite panels and shells [9,10], structures with pressurised shells [11], and complex periodic structures [12–15]. Hussein et al. [16] produced an extensive review of developments in band gap technology.

There are two major mechanisms that have been identified to generate band gap behaviour in periodic structures: Bragg scattering and local resonance. Bragg scattering is observed when a structure exhibits periodic impedance mismatches and the waves are scattered at the borders of the unit-cell (the part of the structure that is periodically repeated). This scattering can be caused by means of inclusions, and geometrical or material inconsistencies, and leads to the interaction of the reflected waves with the incoming waves. When specific circumstances are met, this interaction causes the partial or complete annihilation of wave propagation [16,17]. It can easily be shown that the frequency at which the band gap is observed depends on the length and the material/geometrical mismatch of the unit cell of the periodic structure. This leads to the need for prohibitively large dimensions to achieve low frequency band gaps. Therefore, researchers' focus was shifted to local resonance [18], where a solid core material with relatively high density is usually preferred, suppressed by an elastically soft material. When this sub-wavelength inclusion/addition resonates, it exhibits behaviour that cancels the propagation of waves, giving rise to effective negative elastic constants or group velocities at certain frequency ranges which are significantly lower than those observed in Bragg scattering. Liu et al. [19] examined the transition between the two band gap production mechanisms and there has been research on coupling of the two mechanisms [20,21]. In this work an optimisation method is developed capable of examining both band gap production mechanisms, and the Bragg scattering mechanism is observed in case study geometry to demonstrate its application.

Structures that exhibit band gap behaviour tend to be of

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Nomenclature

β_i	design parameters
f	force vector
q	physical displacement vector
R	transformation matrix
η	loss factor
λ_i	eigenvalue corresponding to frequency f_i
I	identity matrix
M, **K** and **C**	mass, stiffness and damping matrices of the unit cell
m, **k** and **c**	local mass, stiffness and damping matrices of the individual finite elements
p	design parameter vector
x	right eigenvector
$\mathcal{F}(\mathbf{p})$	objective cost function
ω	angular frequency
ρ	mass density
ε_x	propagation constants in the x direction
a_i, b_i, c_i, d_i	design cost coefficients
b_g	band gap
$b\_g_{m\_f}$	band gap mid-point frequency
b_s	stiffener's bending stiffness
E, E_x, E_y, E_z	Young's moduli
f_i	frequency
G_{xy}, G_{xz}, G_{yz}	shear moduli
k_x	wavenumbers in the x direction
L_x	length of the unit cell
l_{top}	length of the top of the stiffener
m	mass
t	time
t_{st}	stiffener's thickness
v_{xy}, v_{xz}, v_{yz}	Poisson's ratios

significantly complex geometry and cannot be realised using conventional manufacturing techniques. Therefore, additive manufacturing (AM) technologies attract an increasing number of researchers [22]. AM eliminates several design limitations and is currently in extensive use in research and industry for small-scale production with a wide variety of manufacturing operations. AM researchers take advantage of the freedom in design that can be attained with relatively low cost, by avoiding the manufacturing cost of additional tools. Matlack et al. [21] examined the broadband vibration absorption of AM meta-structures exhibiting local resonance, while Qureshi et al. [23] modelled and manufactured a cantilever-in-mass metamaterial to achieve wave attenuation. Claeys et al. [24] examined, both numerically and experimentally, several versions of a vibroacoustic metamaterial with local resonance and compared the two sets of results, while Warmuth et al. [25] used AM methods to manufacture a 3D single phase phononic band gap material with cellular design exhibiting a wide stop band at high frequencies. Bilal et al. [26] experimentally observed the trampoline phenomenon on the band gap behaviour of AM metamaterial plates. The wide variety of designs that can be produced using AM methods has led to the opportunity for researchers to examine 3D band gaps in complex structures [27,28]. This wide use of AM has led researchers to examine the effect of the property variability of AM on the experimental results [29,30], where a better agreement was obtained between analysis and experimental results by considering uncertainties in the resonators and the host structure.

Efficient and accurate optimisation schemes are essential when determining design solutions and researchers use several methods. The optimal design of an Euler-Bernoulli simple band gap beam, made of linearly elastic material, has been recently examined [31], where the author used a bound optimisation method which optimised the gap between natural frequencies. Hussein et al. [32–34] optimised the band gap behaviour of periodic layered structures, where methods to achieve band gap behaviour within specific frequencies were developed. More specifically, Hussein et al. employed a multi-objective genetic algorithm that generates a population of possible solutions and searches for the optimum one. This method was tailored so that several objectives were examined, such as the percentage of the band gaps in specific frequencies, low frequency band gaps and control of the speed of energy propagation in the structure. Jensen and Sigmund [35] optimised the band gap behaviour of phononic structures using topology methods. Langley et al. [7] used a quasi-Newton algorithm with 30 random-start runs to obtain the optimal vibration absorption of the structure, while Wormser et al. [27] optimised the phononic band gap behaviour of cellular structure using gradient based methods. To the authors' knowledge, all the optimisation methods for phononic band gap behaviour focus solely on the band gap tailoring itself.

The novelty of the work presented in this paper is:

- An optimal design of a band gap structure is obtained, so that it serves both as a stiffener and band gap production mechanism, constituting a structural part.
- The optimal design of the structure is obtained by applying a developed computationally efficient unit cell based optimisation scheme.
- The developed multi-disciplinary design optimisation scheme uses scalarisation for simultaneous mass and vibration minimisation and static stiffness maximisation. Several starting points are used and parametric analysis is completed to evaluate the optimal solution.

More specifically, the multi-disciplinary optimisation of vibration attenuation through band gap and static structural performance of a 1D composite structure with powder bed fusion of polyamide 12 (PA12) material additions is examined. The structure is modelled using finite element (FE) method and PST is used for predicting its wave

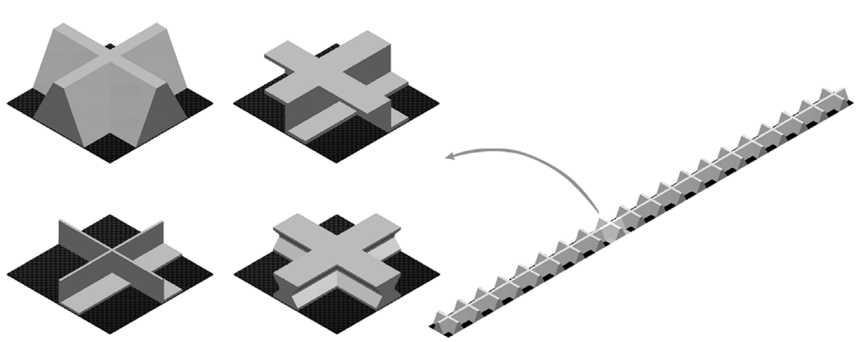

Fig. 1. Several schemes of periodic structures.