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High performance 3-node shell element for linear and geometrically nonlinear analysis of composite laminates

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ABSTRACT

Thin-walled structures hold primacy among modern engineering structures. All the advantages offered by the curved geometry and thinness of the walls come even more to the fore when combined with exquisite properties of fiber-reinforced composite laminates. Directionally dependant material properties open vast possibilities for tailoring global structural properties and, therewith, optimization. Successful design of such structures calls for high performance shell type finite elements. This paper presents a linear triangular shell element based on the equivalent single-layer approach and the first-order shear deformation theory. The shear locking effect is resolved by the descrete shear gap (DSG) approach combined with the cell smoothing technique. To improve the element performance with respect to the membrane behavior, the assumed natural deviatoric strains (ANDES) formulation is applied, with necessary modifications to meet the requirements of curved structures with anisotropic material properties. Geometric nonlinearities are addressed by the co-rotational formulation. Examples demonstrate the element applicability and performance.

1. Introduction

With a roughly estimated share of some 80%, thin-walled structures make the group of most commonly encountered engineering structures. This is clearly the consequence of numerous advantages they offer. The thinness of the walls combined with the curved geometry allows the use of high membrane stiffness to carry transversely applied loads. In this manner, a favorable load-to-weight ratio is achieved. The advantages provided by the geometry are further enhanced through application of modern engineering materials, primarily fiber-reinforced composite (FRC) laminates. FRC laminates provide outstanding mechanical properties combined with further weight saving. Also, directionally dependent properties intrinsic for composites make possible tailoring of structural properties already on the material level.

The research on FRC laminated structures is quite diverse, ranging from the work on their general improvements to the work on failure models, detection and localization. As a possibility for further improvement of already exquisite material properties, a number of researchers considered the use of functionally graded materials [1,2,3]. At the same time, a great potential for improvement of general structural properties was seen in application of multi-functional materials, such as piezoelectric materials, which allow active control of their mechanical behavior [4]. Consequently, a great deal of work was dedicated to the development of modeling tools for active composite laminates [5,6,7,8]. On the other side of the research spectrum, due to the proneness of FRC laminates to hidden failures including delamination, research efforts strived to provide reliable models for interlaminar damage and failure of FRC structures, as presented in the survey by Rohwer [9]. Also, methods were developed with the aim of non-destructive damage detection and localization [10,11]. This rather short glimpse at the research scope related to the composite laminate structures serves only to give a general impression about the attractiveness of the topic.

Accurate and reliable modeling and simulation are the prerequisites of successful research in all the above mentioned research directions. Most frequently the Finite Element Method (FEM), as the method of choice in the field of structural analysis, is used for the purpose. Depending on the research field, models of various complexity and detail levels may be required. This work puts focus onto the global structural behavior. Bearing this in mind, the main workhorse elements in FEM programs are equivalent single-layer shell elements based either on the classical first-order theory (Kirchhoff-Love elements) or the first-order shear deformation theory (Reissner-Mindlin elements). The latter is more general as it includes the consideration of transverse shear effects, which is a rather important aspect for composite laminates. However the main reason why most elements are based on it relies

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primarily on the reduced continuity requirements from the FE shape functions. The family of degenerate shell elements is a large group of shell elements based on the Reissner-Mindlin kinematics and most of them have also been applied for modeling laminate composite structures [12,13,14]. The basic advantage of this element group is the applicability to a relatively wide range of thickness and curvatures. However, they are limited in the aspect of strain and stress recovery in case of laminate structures. Layer-wise theories [15,16], offer a remedy with respect to the stress/strain recovery, but this positive aspect is accompanied by an increased numerical effort. As a well-balanced compromise, Carrera et al. [17] and Valvano and Carrera [18] proposed finite elements with node-dependant kinematics. The approach combines the equivalent single-layer approach and the layer-wise approach. The basic idea is to apply the latter locally in order to provide the adequate accuracy in the structural sub-domains where the strains and stresses are of interest. Finally, the recently proposed isogeometric approach addresses the problem of seamless integration of design and analysis. The basic idea behind it resides in the use of the same shape functions (NURBS) for both the description of CAD geometry and displacement field of the FE model. Isogeometric developments for composite plates and shells involved the Kirchhoff-Love, Mindlin-Reissner and higher order kinematics [19,20,21,22].

Obviously, the development of finite elements for shell structures and particularly those made of composite laminates has attracted a great deal of interest. This paper aims at a high performance triangular shell element based on the first-order shear deformation theory. Geometric nonlinearities are addressed by means of the co-rotational formulation.

2. Triangular shell element

The obvious advantages offered by linear triangular elements are the exquisite meshing ability and very high numerical efficiency regarding the computation of matrices and vectors for a single element. As usual, advantages are accompanied by certain disadvantages. Not only may the convergence rate be rather slow due to the ability of the classical linear 3-node element to represent only constant strain and stress states, but the element is also susceptible to the notorious shear locking possibly causing convergence to an erroneous, stiffer solution. To resolve these issues and produce a high performance element, both the bending and membrane behavior of the element will need to be properly modified.

In the element formulation, both the global (x, y, z) and local (x', y', z') coordinate systems are used, Fig. 1. The local coordinate system is defined so that the z'-axis is perpendicular to the element surface, while one of the in-plane axes, the x'-axis, is oriented from element node 1 toward element node 2.

The element employs the classical linear shape functions:

$$N_1(x', y') = \frac{1}{2A}(x_2'y_3' - x_3'y_2' + y_{23}'x' + x_{32}'y')$$

$$N_2(x', y') = \frac{1}{2A}(x_3'y_1' - x_1'y_3' + y_{31}'x' + x_{13}'y') \tag{1}$$

$$N_3(x', y') = \frac{1}{2A}(x_1'y_2' - x_2'y_1' + y_{12}'x' + x_{21}'y')$$

where x_i' and y_i' $(i = 1,2,3)$, are the local coordinates of the element nodes, A is the element surface area and x_{ij}^{i} and y_{ij}' denote the abbreviated coordinate differences, i.e. $x_{ij}' = x_i' - x_j'$ and $y_{ij}' = y_i' - y_j'$. The element geometry is simply regenerated from its mid-surface:

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \sum_{i=1}^{3} N_i \begin{Bmatrix} x'_i \\ y'_i \\ 0 \end{Bmatrix} + \sum_{i=1}^{3} N_i \frac{h}{2}\xi\{e_{z'}\} \tag{2}$$

where h is the element thickness and ξ the natural coordinate $(-1 < \xi < +1)$ in the thickness direction. As a consequence of the degeneration process (from 3D to 2D) and the assumed Reissner-Mindlin kinematics, the displacement field $\{u', v', w'\}^T$ in the local coordinates is given by:

$$\begin{Bmatrix} u' \\ v' \\ w' \end{Bmatrix} = \sum_{i=1}^{3} N_i \begin{Bmatrix} u_i' \\ v_i' \\ w_i' \end{Bmatrix} + \sum_{i=1}^{3} \frac{h}{2} N_i \xi \begin{Bmatrix} \theta_{y'i} \\ -\theta_{x'i} \\ 0 \end{Bmatrix} \tag{3}$$

where $\theta_{x'}$ and $\theta_{y'}$ are the rotations around the local x'- and y'-axes and i in the right subscript denotes the node number.Up to this point, all the equations fit into the classical formulation. However, as already discussed above, the strain field directly derived using the kinematic relations produces a too stiff element that suffers sub-optimal convergence or even a convergence toward an erroneous solution. Hence, special techniques are needed as a remedy. Since a flat element is considered here, its deformational behavior can be represented as a superposition of plate and membrane elements. The development presented here implements already existing solutions for both bending and membrane behavior, but it represents a novel combination of those solutions. In what follows, the basic ideas and most important formulae are given as the available literature that is referenced below provides the necessary details.

2.1. Plate behavior

Since the element is based on the first-order shear deformation theory, the stiffness matrix of the plate element consists of the bending stiffness and transverse shear stiffness. Using the discretized displacement field (Eq. (3)), the bending strains with respect to the local coordinate system are directly derived:

$$\begin{Bmatrix} \varepsilon_{x'x'}^b \\ \varepsilon_{y'y'}^b \\ \gamma_{x'y'}^b \end{Bmatrix} = z' \begin{Bmatrix} \frac{\partial \theta_{y'}}{\partial x'} \\ -\frac{\partial \theta_{x'}}{\partial y'} \\ \frac{\partial \theta_{y'}}{\partial y'} - \frac{\partial \theta_{x'}}{\partial x'} \end{Bmatrix} = z' \sum_{i=1}^{3} \begin{bmatrix} 0 & 0 & N_{i,x'} \\ 0 & -N_{i,y'} & 0 \\ 0 & -N_{i,x'} & N_{i,y'} \end{bmatrix} \begin{Bmatrix} w_i' \\ \theta_{x'i} \\ \theta_{y'i} \end{Bmatrix} \tag{4}$$

thus yielding the corresponding strain-displacement matrix $[B_{pb}]$ in the

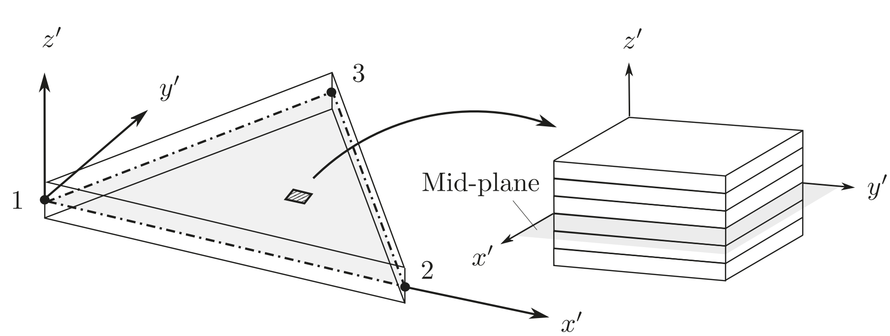

Fig. 1. Element geometry, coordinate systems and material architecture.