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Micromechanics of kink band formation in open-hole fibre composites under compressive loading

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ABSTRACT

The micromechanics of kink band formation in open-hole fibre composites under compressive loading is described. The objective being the development of a methodology for designing of structural components with open-holes. Our results explain why failure by kink band formation propagates from the edges of an open-hole in a direction almost perpendicular to the loading direction and why the 0 degree plies govern the compressive failure of an open-hole laminate. The proposed design methodology accounts for the microstructure, including the fibre/matrix bonding, and the nonlinear behaviour of the constituents, enabling it to prevent local failure at the hole edges, or global failure, by kink banding of a laminate containing stress concentrations.

1. Introduction

The compressive strength of a unidirectional fibre composite is often less than 60 % of its tensile strength; hence, in many cases, the compressive strength dictates the design. An important mechanism of compressive failure in fibre composites is a localized imperfection-sensitive material instability known as plastic microbuckling, leading to the formation of one or more kink bands by localization of plastic deformation. Both terminologies, i.e. plastic microbuckling and kink band formation, are adopted in this paper. Manufacturing and curing of a composite inevitably introduce fibre misalignments. The matrix provides lateral stability to the fibres and prevents, or postpones, the occurrence of plastic microbuckling, inducing a complex strain field in the matrix. After a small amount of fibre rotation under a remote compressive stress, geometric softening associated with the fibre rotation outweighs the plastic strain hardening of the matrix, and a microbuckle nucleates and then propagates through the composite. The remote compressive stress, at which the composite fails by the formation of kink bands, i.e. the critical stress, is sensitive to fibre misalignments since the load carrying capacity of the material is lost locally. The latter description emphasizes the importance of accounting for the nonlinear behaviour of the constituents and the microstructure of a composite when investigating plastic microbuckling. In the early work on kink band formation, Argon [1] and Budiansky [2] identified that fibre misalignments and plastic shear deformation in the matrix govern the critical stress. Christoffersen and Jensen [3] developed a homogenized constitutive model accounting for the microstructure of a unidirectional fibre composite and allowing all cases of time-independent elastic-plastic behaviour of the constituents, making it suitable for analysing plastic microbuckling. Sørensen et al. [4] implemented the model [3] as a user subroutine (UMAT) into the finite element program ABAQUS. Jensen [5] extended the latter formulation to account for full decohesion between the matrix and the fibres. The models in Refs. [3] and [5] do not take the fibre bending rigidity into account; however, the fibre bending rigidity decreases with the fibre diameter and in most real-life applications, fibre composites contain many thin fibres embedded in a matrix. A constitutive model for imperfectly bonded fibre-reinforced polymer composites by Skovsgaard and Jensen [6] describes the transition between these two models. Recently, Skovsgaard and Jensen [7] developed a three-dimensional constitutive model for elastic-plastic behaviour of fibre-reinforced composites inspired by the two-dimensional model [3]. Depending on the given structural component and how well controlled the fibre misalignments are, due to manufacturing, the critical stress may not be suitable for engineering applications. Beyond the critical stress, in the post-critical region, the kink band may propagate (under steady-state conditions) through the material at a constant stress known as the steady-state kinking stress. At steady-state, the fibres in the kink band stop rotating, i.e. lock-up, forcing the kink band to spread into the base material. A semi-analytical model by Jensen [8] predicts the steady-state kinking stress. Recently, Skovsgaard and Jensen [9] developed an analytical model for steady-state kinking. Liu et al. [10] suggest that, for unidirectional fibre composites containing stress concentrations, e.g. holes and notches, a lower bound for the critical stress can be estimated by the steady-state kinking stress. Structural applications of fibre composites may include components with open-holes. The ability to predict compressive failure of open-hole fibre composites is a problem of

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considerable importance to the Aerospace Engineering community. It provides useful information about how a composite performs in an open-hole application and how resistant it is to defects from fabrication or in-service. Open-hole compression tests on fibre composites suggest that the dominant mechanism of compressive failure is plastic microbuckling [11–20], provided that the matrix displays ductile material behaviour. Plastic microbuckling starts progressively from the edges of a hole and propagates in a direction normal to the loading direction. Modelling of compressive failure interactions in an open-hole T300/914 laminate using a homogenized description of the material separated by interfaces reveals, in accordance with the latter experimental observations, that plastic microbuckling starts progressively from the edges of a hole [22]. Experimental studies by Soutis and Fleck [13], Soutis et al. [14–16] and Ahn and Waas [17] suggest that plastic microbuckling, in the 0° plies, governs the compressive failure of an open-hole carbon/epoxy laminate.

Whitney and Nuismer [23] developed an average stress and a point stress failure criterion for notched laminates. The point stress failure criterion predicts failure when the stress at a distance d_0 away from the hole reaches the unnotched strength. The average stress failure criterion predicts failure when the average stress over a distance a_0 reaches the unnotched strength. The distance parameters a_0 and d_0 vary with hole size and stacking sequence among other factors [24]. After observing that plastic microbuckling, in the 0° plies, governs the compressive failure of an open-hole laminate, Soutis et al. [15] developed a cohesive zone model for predicting the open-hole compression strength. In the cohesive zone model, plastic microbuckling initiates when a remote compressive stress multiplied by a stress concentration factor reaches the unnotched strength. A crack replaces the microbuckled zone and when the crack obtains a critical length such that the remote compressive stress attains a maximum value, catastrophic failure takes place. For the simplicity of application, many textbooks adopt the latter failure criteria; however, these criteria depend on an accurate measurement of unnotched strength and compressive fracture toughness, which highly depend on the microstructure. Ahn and Waas [17] defined a global-local approach in which a micro-region around a hole is analysed. Boundary conditions to be applied on the micro-region are obtained from a global homogenous model. Extensions of this method are validated and used by airframe manufacturers to predict the open-hole compression strength of laminates [25-26]. These studies can capture kink band formation, which is the dominant failure mechanism in compression, and damage in the off-axis plies within a laminate.

Despite the many contributions, the micromechanics associated with kink band formation in open-hole fibre composites under compressive loading remains less understood. This paper aims to describe the micromechanics with the objective of developing a design methodology, accounting for the nonlinear behaviour of the constituents and the microstructure of the composite, for designing of structural components with open-holes. It is inspired by [27]. To accomplish the latter objective, the constitutive models in Refs. [3] and [5] are applied using finite element analysis. The semi-analytical model in Ref. [8] enables a verification of the findings in Ref. [10].

2. Constitutive relations

The constitutive relation

$$\dot{t}_{ij} = C_{ijkl} v_{l,k} \tag{1}$$

relates the nominal stress rates $\dot{t}_{ij}$ to the velocity gradients $v_{i,j}$ through the nominal moduli C_{ijkl}. This paper adopts the index notation, the summation convention and Cartesian coordinates. Latin indices, e.g. i, j, k, l, take values 1,2,3 and Greek indices, e.g. $\alpha, \beta, \eta, \gamma$, take values 1,2. Neglecting the incremental volume change of a material, i.e. $v_{k,k} = 0$, the relation between the nominal moduli C_{ijkl} and the instantaneous moduli L_{ijkl} is

$$C_{ijkl} = L_{ijkl} + \tfrac{1}{2}\sigma_{ik}\delta_{jl} - \tfrac{1}{2}\sigma_{il}\delta_{jk} - \tfrac{1}{2}\sigma_{lj}\delta_{ik} - \tfrac{1}{2}\sigma_{kj}\delta_{il} \tag{2}$$

where the instantaneous moduli L_{ijkl} relate the Jaumann rate of the Cauchy stresses $\hat{\sigma}_{ij}$ to the strain rates $\dot{\varepsilon}_{ij}$, i.e.

$$\hat{\sigma}_{ij} = L_{ijkl}\dot{\varepsilon}_{kl} \quad , \quad \dot{\varepsilon}_{ij} = \tfrac{1}{2}(v_{i,j} + v_{j,i}) \tag{3}$$

and consequently, they satisfy the minor symmetries, i.e.

$$L_{ijkl} = L_{ijlk} = L_{jikl} \tag{4}$$

The relation between the Jaumann rate of the Cauchy stresses $\hat{\sigma}_{ij}$ and the nominal stress rates $\dot{t}_{ij}$ is

$$\hat{\sigma}_{ij} = \dot{t}_{ij} - \omega_{jk}\sigma_{ik} - \omega_{ik}\sigma_{kj} - \sigma_{ij}v_{k,k} + \sigma_{jk}v_{i,k} \quad , \quad \omega_{ij} = \tfrac{1}{2}(v_{i,j} - v_{j,i}) \tag{5}$$

where ω_{ij} denote the spin.

3. Constitutive models

For convenience, the notation in Section 2 is adopted in the matrix form

$$\dot{\boldsymbol{t}}_\alpha^c = \boldsymbol{C}_{\alpha\beta}^c \boldsymbol{v}_{,\beta}^c \tag{6}$$

equivalent to Eq. (1), where

$$\boldsymbol{v}^c = \begin{bmatrix} v_1^c \\ v_2^c \end{bmatrix}, \quad \dot{\boldsymbol{t}}_1^c = \begin{bmatrix} \dot{t}_{11}^c \\ \dot{t}_{12}^c \end{bmatrix}, \quad \dot{\boldsymbol{t}}_2^c = \begin{bmatrix} \dot{t}_{21}^c \\ \dot{t}_{22}^c \end{bmatrix} \tag{7}$$

from which it follows that

$$\boldsymbol{C}_{11}^c = \begin{bmatrix} C_{1111}^c & C_{1112}^c \\ C_{1211}^c & C_{1212}^c \end{bmatrix}, \quad \boldsymbol{C}_{12}^c = \begin{bmatrix} C_{1121}^c & C_{1122}^c \\ C_{1221}^c & C_{1222}^c \end{bmatrix},$$
$$\boldsymbol{C}_{21}^c = \begin{bmatrix} C_{2111}^c & C_{2112}^c \\ C_{2211}^c & C_{2212}^c \end{bmatrix}, \quad \boldsymbol{C}_{22}^c = \begin{bmatrix} C_{2121}^c & C_{2122}^c \\ C_{2221}^c & C_{2222}^c \end{bmatrix} \tag{8}$$

Superscript c refers to one of the constituents, i.e. fibre f or matrix m. The x_1- and x_2- axes are assumed to be parallel with and normal to the fibres prior to deformation, respectively. This section presents the constitutive models by Christoffersen and Jensen [3] and Jensen [5], assuming perfect fibre/matrix bonding and no fibre/matrix bonding, respectively, in the form

$$\dot{\boldsymbol{t}}_\alpha = \boldsymbol{C}_{\alpha\beta} \boldsymbol{v}_{,\beta} \tag{9}$$

equivalent to Eq. (6). When superscripts are omitted, the quantities refer to the composite.

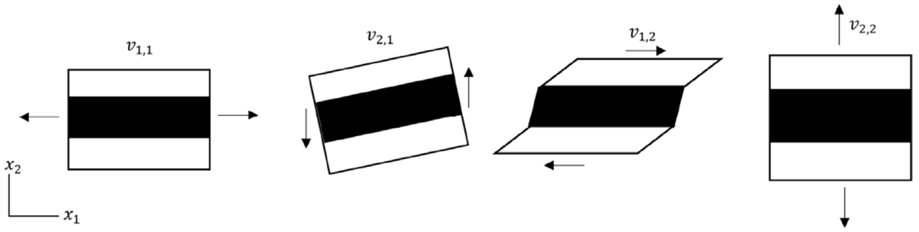

Fig. 1. Incremental deformation in a RVE of a unidirectional fibre composite with perfect fibre/matrix bonding.