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Free vibration and wave power reflection in Mindlin rectangular plates via exact wave propagation approach

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ABSTRACT

Reflection, propagation and energy analysis are crucially important in designing structures, especially plates. A thick plate is considered based on first order shear deformation theory. Wave Propagation Method (WPM) is employed to exactly derive resonant frequencies and wave power reflection from different classical boundary conditions. Firstly, the frequency results are compared with other literature to validate the exact proposed wave solution in the present work. Then, wave analysis and benchmark results for natural frequencies are presented for six different combinations of boundary conditions. The results indicate that the wave power reflection of thick rectangular plates is quite complicated and an incident wave of a specific type gives rise to other types of waves except for simply supported boundary conditions where the reflected wave power does not depend on the system parameters.

1. Introduction

Rectangular plates have extensive application in engineering from Nanotechnology [1] to Aerospace [2] and Biomechanics [3] and many others. Their responses to an external excitation and energy transmission to their neighborhoods must be studied carefully to avoid any probable damage [4].

Many researches have contributed to study the vibration of thin plates such as Leissa's exact solution [5]. Yet, the dynamics of thick plates is quite complex due to the variation of shear deformation across the thickness and effect of inertia forces [6]. Precedent studies in this field done by Reissner [7] and Mindlin [8]. Mindline plate theory, also known as first-order shear deformation theory (FSDT), considers the distribution of shear deformation across the thickness as a linear function and solves the obtained three equations of the motion. Total deflection of the plate consists of bending deflection, shear contribution and angles of rotation. Based on which of the mentioned parameters are considered as fundamental variables, the strategy for deriving the equation(s) of the motion will be determined, considering the fact that reducing the fundamental variables, and consequently equations of the motion will simplify the solution [9,10]. Although numerous analytical and numerical methods have been presented, most of them have limited applications [11,12]. Higher order shear deformation theories (HSDT) involve higher-order expansion of the displacements. This assumption increases the number of unknowns; Murty's theory of HSDT [13] deals with 5, 7, 9 unknowns, Kant [14] with 6 unknowns, and Lo et al. [15] with 11 unknowns.

Numerical and semi-analytical methods come in handy when the complexity of a problem precludes the analytical approaches to be used. The FEM and Rayleigh-Ritz energy methods are two major procedures for solving the obtained equations of the motion. The proposed solutions based on FEM method are able to solve the vibration of moderately thick plates with any combinations of boundary conditions [16]. Yet, shear locking problem is one of their salient concerns due to coupling between bending and shear modes [17]. Recently, Senjanović et al. [18] proposed a shear-locking-free FEM method for vibration analysis of Mindlin plates using bending deflection as a potential function for the definition of total deflection and angles of cross-section rotations. The Rayleigh-Ritz energy method [19–21] and boundary characteristic orthogonal polynomials along with three-dimensional Ritz formulation [22,23] have been used for the free vibration analysis of thick plates with arbitrary boundary conditions. The accuracy of the results is sensitive to the assumed natural modes presented by set of orthogonal functions. Hashemi et al. [24] proposed an exact analytical Levy type solution for thick plates using FSDT. They investigated the free vibration of moderately thick rectangular plates for six combinations of boundary conditions. There are also several other methods for vibration analysis of Mindlin-Reissner plates such as cell-based smoothed radial point interpolation method [25], discrete singular convolution method [26].

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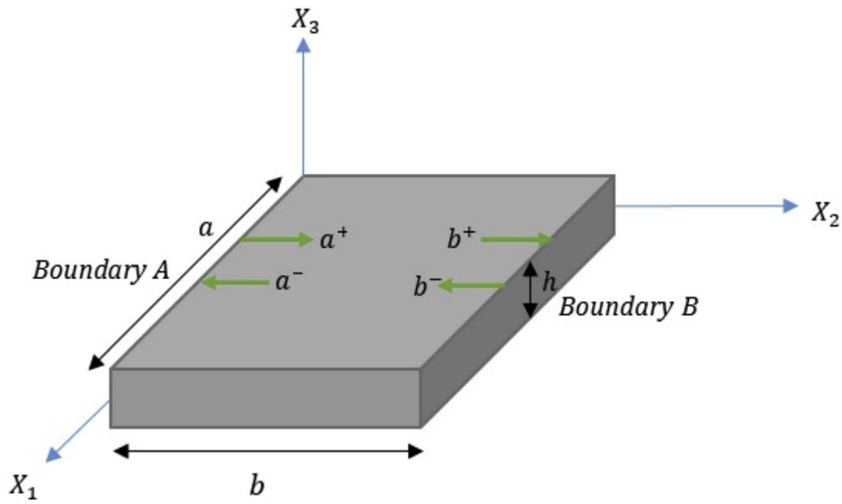

Fig. 1. A Mindlin plate with coordinate convention and waves.

In addition to the methods mentioned above, there exists an exact approach known as wave propagation method (WPM). WPM is a simple, non-iterative, and efficient method for obtaining the natural frequencies of a system. Instead of applying boundary conditions to equations of the motion, reflecting, transmitting and propagating waves will be investigated to determine the natural frequencies and mode shapes of a system. One of the advantages of this method is the ability to study the energy transmission to the neighbors. This is very important for designing structures, because the effect of vibration to the neighbors and bases at frequencies near the natural frequencies will be determined prior to the construction. Wave propagation approach has been utilized mainly for finding the natural frequencies of beams, thin plates, rectangular and circular shells, membranes, frames, Nano-materials, and composite structures. Study of transmission and reflection matrices in Euler-Bernoulli [27] and Timoshenko [28] beams are two cases of WPM method application in beam theories. Bahrami et al. [29] used modified wave approach to find the natural frequencies of non-uniform beams, using Euler-Bernoulli beam theory. In another work, Bahrami et al. [30] used WPM for free vibration of non-uniform rectangular membranes. Annular circular and sectorial membranes were studied in Refs. [31,32] using two dimensional wave propagation. Also, the nonlocal scale effect on buckling, vibration and wave reflection in beams has been studied in Refs. [33,34]. The authors showed that, in nanotubes, the reflected power of an incident wave, except for simply supported boundary condition, is dependent upon the small scale parameter and incident wave frequency. Moreover, Bahrami studied the free vibration, wave power transmission and reflection in multi-cracked nanobeams [35] and nanorods [36]. Furthermore, Bahrami and Teimourian [37] presented the small scale effect on vibration and wave power reflection in circular annular thin nanoplates. Recently, Ilkhani et al. [38] studied energy reflection and transmission in rectangular thin nanoplates. They showed that except for simply supported boundary condition, in other conditions, the obtained coefficients of the transmission matrix, and consequently the energy reflection is dependent on the non-dimensional frequency parameter of the incident wave, the non-dimensional nonlocal parameter, the thickness to length ratio and the number of half waves in length direction.

Reviewing the above acknowledged literature provides us the clue that there is no research conducted on wave analysis and investigation of the effect of thickness of the plate on wave motion, conversion and reflection in thick plates. In all previously done researches in wave analysis of structures, there were at most two waves [27–38] while here there are three waves, and this makes the problem more complicated to analyze. In the present paper, a new analytical approach to analyze the free vibration and wave reflection in thick rectangular plates is presented using wave propagation method. In section 2, the governing equations of the motion with free, simply supported, and clamped boundary conditions are developed; the equations of the motion are rewritten in a specific format compatible with Wave Propagation Method (WPM). In this section, first, the equations of the motion are used to derive the exact propagation matrix, then exact reflection matrices are derived for mentioned boundary conditions. In addition, the propagation and reflection matrices will be helpful for future works that has to do with wave power transmission and reflection in waveguide structures. In section 3, numerical results are presented and investigated. The results are compared with other literature and exact benchmark results are presented for the natural frequency for various aspect ratios, thickness to length ratios, and boundary conditions. As they are considered to be exact results, other researchers can use them to verify their approximate solutions in future works. Finally, the behavior of the reflection matrices is discussed for different boundary conditions. These results are discussed thoroughly for different thickness to length ratios and frequency ranges. Various boundary conditions are also considered to analyze the wave power reflection at boundaries. These results depict the behavior of the reflection coefficients which shows the energy reflected and dissipated at boundaries.

2. Methodology

2.1. Governing equation of motion

The non-dimensional equations of motion based on Mindlin plate theory for a flat, isotropic, thick rectangular plate of length a, width L and thickness h as shown in Fig. 1 are [24]:

$$\widetilde{\psi}_{1,11} + \eta^2\widetilde{\psi}_{1,22} + \frac{\nu_2}{\nu_1}(\widetilde{\psi}_{1,11} + \eta\widetilde{\psi}_{2,12}) - \frac{12K^2}{\delta^2}(\widetilde{\psi}_1 - \widetilde{\psi}_{3,1}) = -\frac{\beta^2\delta^2}{12\nu_1}\widetilde{\psi}_1, \tag{1a}$$

$$\widetilde{\psi}_{2,11} + \eta^2\widetilde{\psi}_{2,22} + \frac{\nu_2}{\nu_1}\eta(\widetilde{\psi}_{1,12} + \eta\widetilde{\psi}_{2,22}) - \frac{12K^2}{\delta^2}(\widetilde{\psi}_2 - \eta\widetilde{\psi}_{3,2}) = -\frac{\beta^2\delta^2}{12\nu_1}\widetilde{\psi}_2, \tag{1b}$$

$$\widetilde{\psi}_{3,11} + \eta^2\widetilde{\psi}_{3,22} - (\widetilde{\psi}_{1,1} + \eta\widetilde{\psi}_{2,2}) = -\frac{\beta^2\delta^2}{12K^2\nu_1}\widetilde{\psi}_3 \tag{1c}$$

where $\delta = \frac{h}{a}$ is the dimensionless thickness to length ratio, $\eta = \frac{a}{b}$ is aspect ratio, $\beta = \omega a^2\sqrt{\frac{\rho h}{D}}$ is the non-dimensional frequency parameter, and K^2 is the shear correction factor to acknowledge the fact that transverse shear strains are not independent of the thickness coordinate. Also, $\nu_1 = (1-\nu)/2$ and $\nu_2 = (1+\nu)/2$ where ν is Poisson's